A Behavioral Study of Capacity Allocation in Revenue Management

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In this paper we present a set of laboratory experiments that investigate how human subjects solve the two-class capacity allocation revenue management problem. The two-class problem is the simplest possible revenue management problem, that is a fundamental building block for a family of complex and sophisticated RM problems. We study the capacity allocation problem with ordered and unordered arrival, as well as a simplified version of the problem in which the decision-maker makes the single capacity allocation decision at the beginning of the selling season. We find that our laboratory participant generally accept too many low class customers, leave to few units of unused capacity, and as a result do not accept enough high class customers. We also find that making decisions up-front improves performance in the ordered arrivals case, and does not hurt performance in the unordered arrivals case. Lastly, we find that in the unordered arrivals setting, participants accept more low class customers than they should, when there is ample capacity, and reject more low class customers than they should when there is scarce capacity.

Key words: Behavioral Operations, Revenue Management, Capacity Allocation, Newsvendor.

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1. Introduction

The practice of Revenue Management (RM) is considered to be one of the biggest success stories of Operations Research. Originated by Sabre (www.sabre.com) in 1970s, it is credited for increasing the revenue gains of the airline industry by 5-7% (Donovan, 2005), and has since been adopted by other industries that share similar characteristics (hotel, car rental, media and broadcasting, cruise lines and ferry lines, theaters and sporting events and so on). The essence of RM is price discrimination—charging different prices to different customers for what is essentially the same product. Product attributes that make RM effective include fixed and highly perishable capacity that requires to be sold in advance of consumption, highly variable demand, and a heterogeneous customer base (Talluri, 2012).

In many industries RM models and software automatically control the bulk of inventory (Talluri and Van Ryzin (2006) describe implementation details for such traditional RM industries as airlines and hotels), but even these traditional users of RM must combine RM software with manual oversight to achieve the best results (Talluri, 2012). This managerial augmentation of automated system is useful due to the presence of inevitable exceptions (special days or events). So RM software does not fully replace human judgment even in traditional industries in which the majority of the decisions are automated. There are also numerous industries that share many of the characteristics that make them candidates for benefitting from some of the RM methods and techniques, which usually do not widely use RM software. Examples include restaurants, spa resorts, golf courses (Kimes, 2003) and many other small businesses that still involve managers in RM decisions. Some of the RM decisions made by these smaller businesses range from accepting appointments for a hair-saloon, to deciding how much beer to deliver at each store along a delivery route (Bassok & Ernst, 1995), to how many bagels to reserve for sandwiches at lunch (Gerchak, Parlar, & Yee, 1985).

At the heart of all these RM problems is the basic tradeoff—to sell a product at a lower price, or to forego this certain revenue by postponing the sale in hopes of higher revenue in the future, while also accepting a risk of leaving the unit unsold. Our goal in this paper is to understand how human decision makers solve the basic RM problem (the single resource independent class problem) that is a fundamental building block for a family of complex and sophisticated RM problems. The goal of our research is three-fold:

1. To characterize behavioral biases that may or may not affect human decision-makers faced with a RM problem.
2. To compare several ways of framing the problem to human subjects in order to establish which ways are more effective than others.
3. To estimate the potential value from using automated RM systems by comparing the performance of human subjects to the performance of the optimal solution.

2. The Models and Settings in Our Study

The analytic side of optimally allocating the capacity of a limited, perishable resource to different customer segments with the goal of revenue maximization is a well-studied problem in the operations management literature. We discuss some of this literature in Section 3. We contribute to the revenue management literature by studying how human decision-makers solve this problem.

We consider the single resource RM problem with two independent customer classes (*low class* has a low willingness-to-pay, and the *high class* has a high willingness-to-pay). Our human subjects (who are in the role of the firm) must find protection levels (i.e., the number of units that must be protected for a particular class) to maximize revenues. Prices are exogenous. We chose this setting because it is the simplest instance of the RM problem and is a basic building block for a general class of models.

The two main treatments in our study are the *ordered arrival* and the *unordered arrival* treatments. In both treatments, for each customer that arrives, the participants, knowing the customer class, decide whether to accept this customer or to turn it away. In the ordered arrival treatment, low class customers arrive first, followed by the high class customers. It is well known that the solution for the ordered arrival two-class problem is to set the protection level for high class customers at

$$y^* = \min\left\{F_h^{-1}\left(1 - \frac{p_l}{p_h}\right), S\right\}$$

where F_h^{-1} is the inverse of the cumulative distribution function for the high class demand, p_l and p_h denote low price and high price, respectively, and *S* denotes the total capacity.

In the unordered treatment, customers arrive in an arbitrary order. This problem is solved with dynamic programming. It is shown in the literature (see e.g., T. C. Lee and Hersh (1993) and Papastavrou, Rajagopalan, and Kleywegt (1996)) that, at stage *t* with *s* remaining units, the optimal policy is a threshold rule:

$$\varphi^*(t, s, p_l) = \begin{cases} \text{Accept}, & \text{if } p_l \geq V_{t-1}^{s} - V_{t-1}^{s-1}; \\ \text{Reject}, & \text{if } p_l < V_{t-1}^{s} - V_{t-1}^{s-1}. \end{cases}$$

The value function (V_t^s) indicates the expected future total revenue that can be generated when there are t time intervals remaining until the end of the selling season and s units on hand. It is computed as follows:

$$V_t^s = (1 - \lambda_h - \lambda_l)\, V_{t-1}^s + \lambda_h [p_h + V_{t-1}^{s-1}] + \lambda_l \max \begin{cases} p_l + V_{t-1}^{s-1} \\ V_{t-1}^s \end{cases}$$

with boundary conditions

$$V_0^s = 0, \text{for all } s, \qquad \text{and} \qquad V_t^0 = 0, \text{for all } t.$$

In the above expression, λ_h and λ_l denote the arrival probabilities for high class and low class customers respectively[1]. The threshold ($V_{t-1}^s - V_{t-1}^{s-1}$) indicates the expected marginal value of the s^{th} unit at time t-1.

In addition to the two main treatments (ordered and unordered arrivals) we conducted a third treatment, in which subjects set protection levels up-front rather than making a decision for each customer as it arrives. This up-front decision format is mathematically equivalent to the ordered arrival treatment (because the two customer classes are independent, so the optimal protection level for high class customers does not change based on the number of low class customers observed). Therefore, comparing the ordered arrival treatment and the up-front treatment will allow us to see the effect of the decision-making format on performance. We note that both of these treatments are also mathematically equivalent to the newsvendor problem in inventory theory, i.e., they yield the same optimal solution when the problem parameters are set accordingly.

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[1] A technique for obtaining arrival probabilities is given by Lee and Hersh (1993).

The up-front decision format will yield sub-optimal solutions to the unordered arrival problem. Therefore, comparing the unordered arrival treatment and the up-front treatment will provide a measurement of the actual revenue loss (if any) from allowing decision-makers to use a simplifying heuristic.

3. Related Literature

Literature describing the theory and practice of RM is quite vast, and we do not attempt a comprehensive review here. We refer the reader to Özer and Phillips (2012) for a comprehensive review of pricing management. The work on static (ordered arrivals) capacity allocation decisions can be found in Littlewood (2005), Belobaba (1989), Curry (1990), Wollmer (1992), Brumelle and McGill (1993), Robinson (1995); and dynamic (unordered arrivals) models are covered in T. C. Lee and Hersh (1993) and Lautenbacher and Stidham Jr (1999). McGill and Van Ryzin (1999) provide a comprehensive survey of the early literature, while Talluri and Van Ryzin (2006) describe more recent developments.

Our study falls into the area of behavioral operations management because we use controlled laboratory experiments with human subjects to test analytical models (see Katok (2011) for an overview of this literature). There are fewer behavioral than analytical studies of problems related to revenue management and dynamic pricing. Bearden, Murphy, and Rapoport (2008) consider a firm selling a fixed number of units and facing uncertain demand for the product. In their setting, each customer has a different willingness to pay for the product and makes a price offer to purchase a unit of the product. Firm needs to decide whether to accept or reject an arriving offer. The authors try to understand what kind of decision policies (sophisticated policies vs. simple heuristics) the decision makers employ while making RM decision. Our problem studied in unordered arrivals setting is similar to Bearden et al. (2008)'s setup, however it also differs from it in several aspects. Their problem considers multiple (to be precise infinite) customer segments and mainly aims to investigate the behavior in pricing RM decisions, while we focus on a problem with two customer segments and aim to investigate the behavior in capacity allocation RM decisions. In addition, they study the problem in unordered arrivals setting, whereas we consider our problem in both ordered and unordered arrivals settings. Our aim with this design is to build a bridge between simple (ordered arrivals) and sophisticated (unordered arrivals) RM models, which can enable us to understand the human behavior thoroughly, starting from a simple model and building on it gradually.

Bendoly (2011) examines the physiological responses of decision makers engaged in RM tasks. The author matches Bearden et al. (2008)'s design and varies the number of tasks that are simultaneously presented to decision makers. The author argues that disparity between actual and normative behavior are due to arousal and stress associated with RM tasks.

Kocabıyıkoğlu, Göğüş, and Gönül (2014) study the two-class RM problem and a closely related newsvendor problem, and test whether the behavior in these two mathematically equivalent models are equal in the laboratory. Their results indicate that decision makers do not perceive and process these two models as identical. They find that subjects' RM decisions (protection levels) are consistently higher compared to the newsvendor order quantities. The authors argue that minimizing unsold units becomes more salient in the newsvendor problem, and this may explain why subjects order fewer units in newsvendor problem than the RM problem.

The newsvendor problem – which is closely related to the ordered arrival two-class RM problem– is one topic that has received a good deal of attention in the behavioral operations literature. Recent papers typically take one of the following four approaches: (i) document the biases in newsvendor behavior, see e.g., Schweitzer and Cachon (2000), Bolton and Katok (2008), (ii) propose different arguments to explain the behavior, see e.g., Su (2008), Ren and Croson (2013), Ho, Lim, and Cui (2010), (iii) try to improve performance by using interventions, see e.g., Y. S. Lee and Siemsen (2013), or (iv) focus on developing a best-response policy given the estimated newsvendor behavior, see e.g., Becker-Peth, Katok, and Thonemann (2013).

Apart from the analysis of seller's behavior in the RM problem, there are several behavioral studies investigating buyers' behavior in this context, specifically studying strategic customer behavior; see, e.g., Kremer, Mantin, and Ovchinnikov (2013), Osadchiy and Bendoly (2010), Mak, Rapoport, Gisches, and Han (2014), Li, Granados, and Netessine (2011), Hendel and Nevo (2006), Nair (2007).

The remainder of the paper is organized as follows. We discuss the details of our experimental design and laboratory implementations in Section 4. We provide theoretical benchmarks and formulate our hypotheses in Section 5. Section 6 presents the experimental results and we conclude in Section 7.

4. Experimental Design

As mentioned in Section 2, our study includes three experimental treatments: (1) Ordered arrivals, (2) Unordered arrivals, and (3) Up-front decision.

In all three treatments we told participants that they are in the role of a manager at a company that sells tickets for an upcoming event (e.g., Broadway show). In all treatments the capacity is $S = 10$, the price for high class customers is $p_h = 200$, and the price for low class customers is $p_l = 20$. High class customers arrive according to $D_h \sim \text{Pois}(5)$, and the low class customers arrive according to $D_l \sim \text{Pois}(15)$. Participants are required to allocate the 10 units of available capacity (tickets) between the two customers.

To ensure that our three treatments are perfectly comparable, we keep the expected number of low-type and the expected number of high-type customers constant across the three treatments. The solution for the ordered arrival and up-front treatments require an estimate of the probability distribution for the high-type demand, while the unordered arrival treatment requires the knowledge of the arrival probabilities for both high-type (λ_h) and low-type (λ_l) customers in each interval t[2]. To make the three treatments comparable, we keep the underlying demand distributions for each customer type the same in all three treatments. To achieve this, we choose Poisson distribution to model the demand, and utilize 'Poisson to binomial approximation' to obtain arrival probabilities for the unordered arrival treatment. We chose these specific demand and capacity parameters to ensure that RM decisions are relevant (i.e., capacity is binding).

At the beginning of each decision round participants see capacity, price and demand information.[3] Also, in all three treatments participants know that there will be at least 10 low

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[2] We use fixed arrival probabilities for each *t*, thus instead of λ_h(t) (resp., λ_l(t)), we use the notation λ_h (resp., λ_l).
[3] The vector of actual demand was determined prior to the experiment and was the same for every subject.

class customers. Demand information that participants observe and the feedback information they see at the end of each round, vary across treatments.

In ordered and unordered arrival treatments, we tell the participants that each round of the game consists of multiple periods. In each period, a customer arrives with a certain probability. In the ordered arrival treatments we tell them that low-type customers arrive first, and high-type customers arrive after all low-types have arrived. In the unordered arrival treatment we tell participants that each period a low class customer arrives with probability $\lambda_l = 0.0244$, or a high-type customer arrives with probability $\lambda_h = 0.0083$ (but never both). We also provide the participants with the probability distributions (displayed graphically and described as "Poisson" with the mean) for the total number of both high-type and low-type[4] customers.

Each round consists of 600 decision periods. Whenever a customer arrives, participants observe the class of the customer (high or low), remaining capacity in the current round, (the remaining number of periods in the current round in unordered arrivals treatment), and capacity that has already been allocated to high class and to low class customers, and her revenue earned so far in the current round. Then the participant is asked to either accept or reject the new arrival. Once a round ends (which happens either when all 10 units of capacity have been allocated or when the last customer has arrived), participant see feedback information on how many units were allocated to each of the customer classes, realized high-class and low-class demands, the unsold capacity, and the resulting total revenue for this round.

[4] In ordered arrival treatment, subjects observe low-type arrivals before they start to observe high-types, thus we also provide them low-type demand information, although it is not required by the optimal policy.

In the up-front treatment, we tell the participants the probability distribution for high-class demand, again, described as "Poisson" with the mean and displayed graphically. To avoid providing participants with irrelevant information, we simply tell them that low-class demand will be at least for 10 units (and do not show them low class demand distribution). At the beginning of each decision making round, participants decide how many of the 10 units of capacity they want to reserve for high-class customers. After the decision is made, participants see feedback information on the outcome of the round: the number of units reserved for high class customers, realized high-class demand, the resulting unsold capacity, and her resulting revenue for this round.

In total, 75 human subjects participated in our study: 24 in the ordered treatment, 26 in the unordered treatment, and 25 in the up-front treatment. Each person participated in a single treatment only (this is a between subject design). In all treatments, upon arrival to the laboratory, the participants were randomly seated in visually isolated cubicles and were provided written instructions that describe the rules of the game, the use of the software, and the payment procedures (see Appendix A for the instructions used in the unordered treatment; instructions for other treatments are similar and available upon request). After all participants had a chance to read the instructions on their own, the experimenter read the instructions to them aloud, invited participants to ask questions, and answered any questions before starting the session. The same experimenter conducted all sessions in this study. Experiments consisted of 40 decision rounds (with ordered and unordered arrival treatments having multiple decisions in each round), thus each subject played the game 40 times. In all three treatments, as the rounds progressed, subjects received historical information about the outcomes in prior rounds of the game.

All treatments were conducted at a major U.S. public university. Participants were recruited through the on-line recruitment system ORSEE (Greiner, 2004). The experimental interface was programmed using the zTree system (Fischbacher, 2007) for all treatments. The snapshots of the computer screens for the unordered arrival treatment are provided in Appendix B.

Subjects made their decisions and received experimental currency units (ECUs) based on the revenues they earned. Cash was the only incentive offered to the participants. The total ECUs across all 40 rounds were converted to cash payments at the end of the session at the rate of 3000 ECUs = $1 US. The average earnings for all treatments were $18.01 US dollars, including a $5 US dollars participation fee for each subject. Sessions lasted between 50 and 80 minutes.

5. Theoretical Benchmarks and Research Hypotheses

Table 1 summarizes theoretical benchmarks for optimal solutions for the three treatments in our study. We formulate research hypotheses based on these theoretical benchmarks.

Table 1. Theoretical Benchmarks

Treatment	High class customers served	Low class customers served	Unused capacity	Average revenue
Ordered Arrivals	4.800	2.000	3.200	1000.00
Unordered Arrivals	4.850	4.075	1.075	1051.50
Up-Front	4.800	2.000	3.200	1000.00

Hypothesis 1: *In all three treatments average performance will not be significantly different from theoretical benchmarks in* Table 1.

Hypothesis 2: *Assuming ordered arrivals, the performance in the ordered arrivals and the up-front treatment will not be significantly different.*

Hypothesis 3: *Assuming unordered arrivals, the performance in the up-front treatment will be worse than in the unordered arrivals treatment.*

6. Experimental Results

6.1 Summary Statistics

Table 2 presents descriptive statistics for the number of both high-class and low-class customers served, unused capacity, and the resulting revenues earned in three treatments, including the mean, and the standard errors (in parentheses). We use the Wilcoxon Signed-Rank Test to compare each entry in Table 2 with its corresponding theoretical benchmark (Table 1) -- taking averages over 40 rounds per subject as the unit of analysis -- and indicate the ones that are significantly different from its theoretical benchmark by an asterisk in Table 2.

Table 2. Descriptive Statistics

Treatment	High class customers served	Low class customers served	Unused capacity	Average revenue
Ordered Arrivals	4.194* (0.069)	4.001* (0.243)	1.805* (0.184)	918.77* (9.593)
Unordered Arrivals	4.340* (0.066)	4.357 (0.183)	1.303 (0.143)	955.21* (10.862)
Up-Front	4.525* (0.059)	2.970* (0.265)	2.505* (0.214)	964.40* (7.564)

Notes. Entries marked with an asterisk are significant at the 0.05 level. The standard errors are in parentheses.

The results show that in all three treatments, participants earn significantly lower revenues than the corresponding optimal levels ($p < 0.001$, for all treatments). Thus, we reject Hypothesis

1 and conclude that in all three treatments, average performance is significantly lower than theoretical benchmarks in Table 1.

We further analyze the revenue results and quantify the amount of loss in revenues for each treatment. Note that without making any capacity allocation decision (i.e., setting protection level at zero), participants are guaranteed to earn a certain amount of revenue which we refer as the *base revenue*. Thus, the base revenue is the revenue earned by accepting first 10 arriving customers. For ordered arrivals and up-front treatments, these customers are known to be low-type, whereas for unordered arrivals treatment, they can be either low-type or high-type. For each treatment, we calculate the average base revenue across 40 rounds and provide it in Table 3 below. Then, we calculate the revenue gains (i.e., additional revenue) generated by capacity allocation decisions in each treatment and provide the % deviation of this measure from the revenue gains achieved by its optimal policy (Table 1). We summarize these results in Table 3. For instance, consider ordered arrivals treatment. Average revenue earned by participants in this treatment is 918.77, and the optimal policy achieves average revenue of 1000 (see Table 1). The average base revenue for the treatment is 200. Thus, revenue gains achieved by participants are 918.77-200=718.77 whereas revenue gains achieved by the optimal policy are 1000-200=800. Therefore, the percentage deviation of the revenue gains from the optimal revenue gains is $\frac{800-718.77}{800} = 10.15\%$.

Table 3. Revenue Losses

Treatment	Average Revenue	Average Base Revenue	Revenue Gains	% Dev. from the Optimal Gains
Ordered Arrivals	918.77	200.00	718.77	10.15%
Unordered Arrivals	955.21	591.50	363.71	20.93%
Up-Front	964.40	200.00	764.40	4.45%

We see from Table 3 that for the treatments ordered arrivals, unordered arrivals, and up-front, the participants fail to capture, respectively, 10.15%, 20.93% and 4.45% of the revenue gains that are captured by the corresponding optimal policy.

To better understand why actual revenues deviate from optimal, we compare the results for number of high-class and low-class customers served, and unused capacity of each treatment to the theoretical benchmarks given in Table 1. This comparison provides an overview of how participants allocate the capacity relative to the optimal policy. For ordered arrivals treatment, the results indicate that participants allocate less units of capacity to high-class and more units of capacity to low-class customers (in other words, they start turning away low-class customers too late), while leaving fewer units of unused capacity compared to the optimal level. In this case, although subjects allocate more units of capacity to low class and therefore sell more units of capacity in total, than the normative level, the revenue gains from these allocations are not enough to compensate the revenue losses resulting from the allocation of fewer units to high-class customers. Thus, participants end up earning significantly lower revenues than the optimal revenue (918.77 vs. 1000). Similar observations are also valid for up-front treatment, although the average numbers of units of capacity allocated to high and low-class customers are closer to optimal in the up-front than in the ordered arrivals treatment. Figure 1 provides a visual summary of these results by showing how the capacity is used across ordered and up-front treatments as well as their corresponding normative level.

For unordered arrivals treatment, the results reveal that participants allocate fewer units of capacity to high-class (similar to ordered and up-front treatments). However, they do not allocate more units of capacity to low class and do not leave more units of unused capacity than the normative level. Thus, participants also end up earning significantly lower revenues than the optimal level in the unordered treatment (955.21 vs. 1051.50). Figure 2 provides a visual summary of how the capacity is used across unordered treatment and its corresponding normative level. Also for comparison, we display in Figure 2 the performance of the up-front treatment under the assumption of unordered arrivals (we label this "Up-Front(U)").

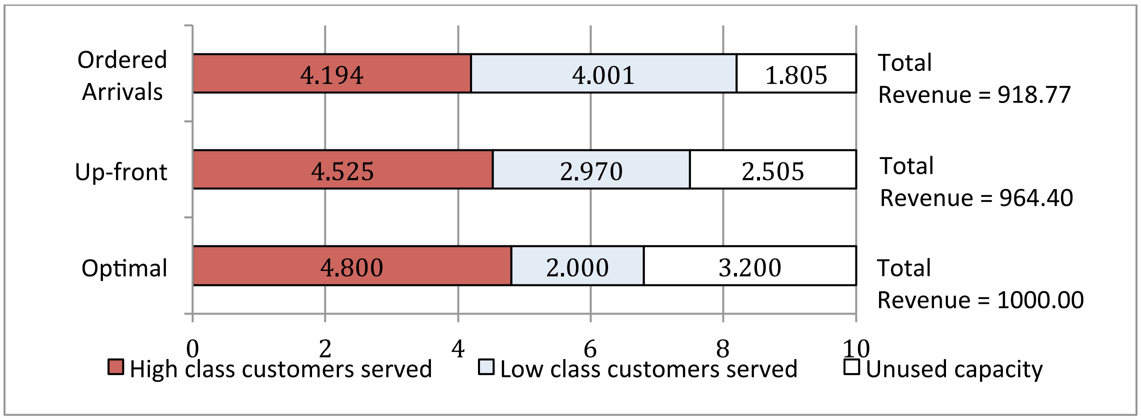

Figure 1. Capacity allocation in the ordered and the up-front treatments and corresponding optimal prediction.

Figure 2. Capacity allocation in the unordered treatment and corresponding optimal prediction.

6.2 The Effect of Decision Format

We compare the results in Table 2 across ordered arrivals and up-front treatments to test our Hypothesis 2, by using Mann-Whitney U Test (Siegel (1956), pp.116-127). Our results reveal that all entries in Table 2 are significantly different across the two treatments, at the 0.05 significance level. As a result, we also reject Hypothesis 2, and conclude that the participants perform better when they make an up-front decision for the entire capacity rather than sequential accept/reject decisions.

Identifying possible causes for the difference in observed behavior between two treatments may help us to improve decision making performance for the unordered arrival setting, in which decisions are also made sequentially. We provide the following arguments as possible explanations for the difference. The psychological cost of rejecting a low-class customer might be greater in sequential decision making (since an individual observes a low-class first, and then rejects the certain revenue she brings) than in up-front decision making (an individual makes a rejection decision for a hypothetical low-type), which may increase the accepted number of low-class customers who arrive first, thereby leaving less capacity for high-class customers who arrive later.

Another possible explanation is that, when an individual makes sequential decisions, she may respond to distorted information (e.g., current allocated capacity to high class or low class), some of them being irrelevant for decision making (e.g., low class demand information), which may also cause the behavior to deteriorate. Furthermore, an individual needs to make multiple decisions (being subject to an error in each of them) in sequential decision making which might increase the complexity and therefore the error rate of overall decision process. The perception of increased complexity might create a higher pressure for decision makers to follow some basic heuristics in sequential decision making than in up-front decision making. To improve individuals' decisions, we must understand the decision-making heuristics that they consult. Recall that a participant receives history information on the realized demand and unsold capacity she had at the end of a decision making round. Thus, it is likely that this information arises as possible anchors for individuals. Next, we examine this conjecture more formally.

- *Demand Chasing*: Demand chasing heuristic assumes that a decision maker anchors on a prior decision and systematically adjusts it in the direction of previous round's demand (Schweitzer & Cachon, 2000). To examine adjustments for the demand chasing heuristic, we use subjects as the unit of analysis and separate out individuals whose *decisions* are statistically correlated with the previous round's high-demand (Lau & Bearden, 2013). In ordered arrival treatment, participants decide on the number of low class customers to be accepted. Thus, we take the correlation between previous round's high-demand draw and current round's number of accepted low class customers for this treatment. We find that 37.5% [9 out of 24] of the subjects' decisions are negatively correlated[5] with the previous round's high-demand draw in the ordered arrivals treatment. In up-front treatment, participants decide on the protection level, thus we take the correlation between previous round's high-demand draw $D_h(t+1)$[6] and current round's protection level $y(t)$ for this treatment. We find that only approximately 12% [3 out of 25] of the subjects' decisions

[5] At the 0.05 level of significance.
[6] Note that our time index t counts backwards, thus previous round is represented by (t+1).

are positively correlated[7] with the previous round's high-demand draw in up-front treatment.

Unused Capacity Aversion: It is also possible that a decision maker may particularly dislike leaving unused capacity at the end of a selling season. Thus, unused capacity can cause, for example, regret, which causes individuals to protect fewer units for high-class customers in the future. To examine adjustments for unused capacity aversion (or the salience of unused capacity) on individuals, we separate out individuals whose *decisions* are statistically correlated with the previous round's unused capacity. Thus, in ordered arrivals treatment, we take the correlation between previous round's resulting unused capacity and current round's number of accepted low-type customers. We find that 20.8% [5 out of 24] of the subjects' decisions are positively correlated[8] with the previous round's resulting unused capacity in ordered arrivals treatment. In the up-front treatment, we take the correlation between previous round's resulting unused capacity and current round's protection level. We find that only 8% [2 out of 25] of the subjects' decisions are negatively correlated with the previous round's resulting unused capacity in up-front treatment.

The results above indicate that both demand chasing and unused capacity aversion[9] are more prevalent among subjects when they make sequential decisions, with demand chasing weighing more. One implication of this observation is that making an up-front decision might help to

[7] At the 0.05 level of significance.
[8] At the 0.05 level of significance.
[9] These two might be correlated, but they still capture different tendencies.

mitigate the adverse effects of the demand chasing as well as the aversion to unused capacity, and therefore can improve the decision making performance.

6.3 Up-front Heuristic for Unordered Arrivals

The results of Section 6.2 reveal that human decision makers employ demand chasing and unused capacity aversion heuristics more when they make sequential accept/reject decisions. Since, unordered arrivals setting also requires sequential decision making, we examine adjustments for demand chasing and unused capacity aversion for the unordered treatment as well.

- *Demand Chasing*: We take the correlation between previous round's high-demand draw and current round's number of accepted low class customers. We find that 38.5% [10 out of 26] of the subjects' decisions are negatively correlated[10] with the previous round's high-demand draw.

  Unused Capacity Aversion: We take the correlation between previous round's resulting unused capacity and current round's number of accepted low class customers. We find that 18.2% [5 out of 25] of the subjects' decisions are positively correlated[11] with the previous round's resulting unused capacity.

These results suggest that individuals have even higher tendency to adjust their behavior on prior demand as well as prior unused capacity in the unordered arrivals treatment than in ordered arrivals treatment, which is intuitive since the problem complexity is higher in the earlier.

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[10] At the 0.05 level of significance.
[11] At the 0.05 level of significance.

We take the next logical step and analyze whether making an up-front decision – that yields sub-optimal solutions to the unordered arrival problem – improves participants' performance. In this section, we compare the results of up-front treatment under unordered arrivals assumption with unordered arrivals treatment. This will provide a measurement of the actual revenue loss (if any) from allowing decision-makers to use a simplifying heuristic.

We note that the results presented for up-front treatment in Section 6.1 assumes ordered customer arrivals, in which the firm fills all units of capacity that are not protected for high class with low class customers. To provide a meaningful comparison between unordered arrivals treatment and up-front decisions, we calculate the performance of the upfront treatment decisions[12] using unordered arrival data, in which any high class customer, arriving when there is still capacity available, will be accepted even if the allocated units for this class exceeds the protection level that has been specified up-front. Thus, revenues for the up-front treatment under unordered arrivals assumption are higher than under the ordered arrivals assumption. We refer the performance of the up-front decisions under the unordered arrival assumptions as Up-Front(U) and provide them in Table 4.

[12] We assume that when individuals make an up-front decision for the unordered arrivals setting, they will set the same protection level as they did in up-front treatment. This assumption is not critical to our analysis and it is reasonable, because in the experiment of up-front decision making in unordered arrivals setting, participants could be provided with additional information that is irrelevant; customers can arrive in arbitrary order instead of low-class arriving before high class, which does not matter, because the decision is made up-front.

Table 4. Comparison of Unordered Treatment with Up-Front(U)

Treatment	High class customers served	Low class customers served	Unused capacity	Total revenue
Optimal	4.850	4.075	1.075	1051.50
Unordered Arrivals	4.340*	4.357	1.303	955.21*
Up-Front(U)	4.533*	2.962*	2.505*	965.84*

Notes. Entries marked with an asterisk are significant at the 0.05 level.

We first compare the results of Up-Front(U) treatment to the results of optimal policy (recall that our analysis in Section 6.1 revealed that participants earn significantly lower revenues in unordered treatment than the corresponding optimal level). The entries that are significantly different from the optimal policy are indicated by an asterisk in Table 4 (for completeness, we again provide the results for unordered treatment in Table 4). The results show that all entries for Up-Front(U) are significantly different from the theoretical benchmarks. We note that neither in unordered arrivals nor in Up-Front(U) treatments, participants reach the normative revenue level.

Next, we compare the results of Up-Front(U) to the unordered treatment. Our analysis shows that the revenues across these two treatments are not different at the 0.05 level (contrary to Hypothesis 3), the average number of high-class customers served is higher in the Up-Front(U) than in the Unordered treatment, the average number of low-class customers served is lower in the Up-Front(U) than in the Unordered treatment, and the amount of unused capacity is higher in the Up-Front(U) than in the Unordered treatment. So it looks like the additional revenue from accepting more high-class customers in the Up-Front(U) is roughly cancelled out by the loss of revenue from accepting fewer low-class customers and leaving more capacity unused. As a result, the overall revenue is the same.

In summary, in the unordered arrivals setting, results show that making an up-front decision results in higher revenues than sequential decisions, but the difference is not significant (p = 0.5912). This indicates that there might be a room for improving decision making performance in unordered arrivals setting. Up-front decision making significantly improves the revenues in ordered arrivals setting, and does not hurt the revenue in unordered arrivals setting, even though restricting decision-makers to a single up-front decision is a significant simplification.

6.4 Further Analysis for Unordered Treatment

Finding optimal policy for the unordered arrivals requires sophisticated decision policies. Considering that humans may not be fully rational, especially decision tasks are complex, the fact that our subjects deviate from the optimal policy is not surprising. In this section, we investigate the behavioral biases -if any- human decision makers exhibit while making dynamic RM decisions.

Making decisions when a high-class customer arrives is trivial, i.e., all high-class customers should be accepted while capacity remains[13]. Thus, for the rest of the analysis in this section, we focus on the decisions that are made for the low-class customers. The optimal policy instructs that a low class customer should be accepted if the revenue from accepting it is at least as high as the opportunity cost of leaving this unit of capacity unused in hopes of allocating it to

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[13] In unordered arrivals treatment, a subject (on average) rejected 1.72% of the high-class customers that she observed.

a high-class customer in the future. Thus, for each decision, the participants are subject to one of the following two errors[14].

- *Accept error*: Rejecting a low class customer that should have been accepted. Thus, the implied revenue loss (or opportunity cost) for an accept error is: $L_{\text{acc}}(s,t) = p_l - R_t^s$.
- *Reject error*: Accepting a low class customer that should have been rejected. Thus, the implied revenue loss for a reject error is: $L_{\text{rej}}(s,t) = R_t^s - p_l$.

Similar to the analysis conducted by Bearden et al. (2008), we examine which type of error is *more common*, which tends to be *more costly*, and *under what conditions these errors are most likely to occur*. To test if the mean implied revenue losses for accept and reject errors are equal, for the same subject, we take the mean implied revenue loss difference between accept and reject errors. Then, we use Wilcoxon Signed-Rank Test to compare if these differences equal to zero. The results show that mean implied revenue losses are significantly higher for reject errors than accept errors (p = 0.0230). This indicates that the participants lose more *revenue opportunities* by myopically accepting a low class customer who should have been rejected. We find no significant difference in number of accept vs. reject errors (p = 0.3344).

To formally compare the effects of remaining time t and remaining capacity s on tendencies to make accept or reject errors, we separately consider observations for which the optimal decision was to “accept”, and observations for which the optimal decision was to “reject”. Note that for the observations for which the optimal decision was to “accept”, the participants are subject to accept errors only. Similarly, for the observations for which the optimal decision was

[14] To be consistent with the literature, we use the same terminology as Bearden et al. (2008) and Bendoly (2011).

to "reject", the participants are subject to reject errors only. We fit the following logistic regression models (with random effects[15]), respectively, to these observations:

$$\mathbf{logit}(Error_{\text{acc}}) = \beta_0 + \beta_t t + \beta_s s + \beta_{ts}(t \times s)$$

$$\mathbf{logit}(Error_{\text{rej}}) = \beta_0 + \beta_t t + \beta_s s + \beta_{ts}(t \times s)$$

where the dependent variables $Error_{acc}$ and $Error_{rej}$ equals 1 if there is a corresponding error and 0 otherwise.

These models allow us to estimate the effects of *t* and *s* (and their interaction) on the probability of making each type of error. Table 5 presents estimation results.

Table 5. Estimates of the Logistic Regression Models for Accept and Reject Errors

Variable	Accept Error	Reject Error
t	0.0018 (0.0017)	-0.0034* (0.0005)
s	-0.1262* (0.0382)	0.1415* (0.0287)
$t \times s$	0.0010* (0.0002)	-0.0001* (0.0001)
Constant	-0.7505* (0.2630)	-0.5320* (0.1724)
Log likelihood	-2213.30	-5768.24
No. of obs. [groups]	3900[26]	10763[26]

Notes. Entries marked with an asterisk are significant at the α= 0.05 level. The standard errors are in parentheses.

We draw the following conclusions from Table 5. The participants are more likely to make an accept error when remaining capacity is lower than when the remaining capacity is

[15] Random-effects models are used to address individual heterogeneity in all our regression results.

higher. We find no systematic relationship between remaining time and accept errors. In addition, the participants are more likely to make reject errors when remaining capacity is higher than when it is lower, and when remaining time is lower than it is higher. In other words, we find that participants tend to be too willing to accept a low class customer when remaining capacity is high, while being too willing to rejects a low class customer when remaining capacity is low. In addition to these observations, we also find that participants tend to be too willing to accept low class customers when remaining time is low. We conclude that departures from optimality are affected both by inappropriate sensitivity to remaining capacity and remaining time.

7. Conclusion

While there exists a considerable body of analytical revenue management research, behavioral studies are fewer. We report on a behavioral study of the two-class capacity allocation RM problem. This is, arguably, the simplest RM problem, and is a basic building block for more complex problems. Thus, a good understanding of how human subjects solve this problem is an important step towards being able to predict and evaluate more complex settings. We first consider the two-class problem with ordered and unordered arrivals, and then introduce a simplified version of the problem in which protection levels are set up-front. The results of our laboratory experiments show that participants significantly deviate from the corresponding optimal policy under all settings we study. These deviations result in a failure to capture revenue gains that can be captured by the optimal policies; losses range from 4% to 21%. These numbers can provide a projection of the potential loss in current systems that do not utilize any RM software.

We identify and report several behavioral regularities. In the presence of ordered customer arrivals, the decision-making format should not matter. However, our results show that the decisions in up-front vs. ordered arrival treatment are systematically different. Participants make better decisions when they make an up-front decision rather than sequential decisions for the entire capacity. Further analysis of our results reveals that participants employ decision-making heuristics more in sequential decision making than in upfront decision making. They tend to adjust their decision by anchoring on previous period's demand (demand chasing) and previous round's unused capacity (with the aim of minimizing unused capacity). A possible managerial implication is that making an upfront decision might lessen the use of heuristics, and help to mitigate the adverse affects of demand chasing as well as the aversion to unused capacity, and therefore can result in better RM decisions.

Similar to the ordered arrivals treatment, decision making is also sequential in the unordered arrivals treatment. Thus, we examine adjustments for demand chasing and unused capacity aversion for the unordered treatment as well, and find that participants have even higher tendency to employ decision-making heuristics in this treatment than in ordered arrivals treatment. Based on the implication (i.e., making an upfront decision can help to mitigate the negative effects of demand chasing and unsold capacity aversion), we take the next logical step and compare upfront vs. sequential decision making in the unordered arrivals setting to find out whether making decisions up-front will also help subjects in this more complex environment. Of course in theory, making an upfront decision results in sub-optimal solutions to the unordered arrivals problem, so we might also expect that in the unordered arrivals setting, the performance when decisions are made-up front might be worse than when decisions are made dynamically. We find that, contrary to the theoretical prediction, up-front decision making results in higher revenues than sequential decision making (although the difference is not significant). However, an implication of this finding is that there is a room for improving decision making process in a dynamic setting, because making an up-front decision is a substantial simplification for the unordered arrival setting which requires the update of the decisions based on remaining time and capacity.

In the presence of unordered arrivals, we observe the following tendencies on participants' behavior. They tend to be too willing to accept low class customers when remaining capacity is high, and too willing to reject low class customers when remaining capacity is low. In addition to these results, participants also tend to demand too little when remaining time is short. We conclude that departures from optimality is affected both by inappropriate sensitivity to remaining capacity and remaining time. We believe that documenting such biases itself is important. Awareness of these biases might enable mangers to be forewarned about them, which can help RM managers to reduce the negative effects of the biases.

We mentioned that ordered arrivals and up-front treatment are mathematically equivalent to the newsvendor problem in inventory theory, i.e., they yield the same optimal solution when the problem parameters are set accordingly. One ancillary observation our study makes is that we find that pull-to-center bias – reported by behavioral studies of the newsvendor problem – persists in RM decisions. Thus, we affirm that the results of newsvendor are somewhat robust to the decision making frame in operations management contexts (i.e., revenue management vs. inventory decisions). One implication of this finding is that the newsvendor results documented in behavioral operations literature might be helpful to understand, and even improve the performance of decision making in capacity allocation decisions. Our results also imply that the decision making format (up-front vs. sequential) may matter for the newsvendor problem also.

Our study reports on several shortcomings of the decision making process in RM decisions. Overall, we believe that the insights of this study can serve to understand and improve decision making performance of RM managers. There are a number of promising avenues for future research. First, we take for granted that demand for each customer segment is independent of the (capacity) controls being applied by the firm. That is, a denied customer will not upgrade to a high class (i.e., buy-up), or she will not downgrade to a low class (i.e., buy-down) and is simply lost. Incorporating buy-down and/or buy-up behavior can have important consequences on the revenues, and thus seems worthwhile to examine. In addition, understanding how competition affects RM decisions, when human decision makers are involved, would be quite helpful for firms. Thus, looking into a model in which two (or more) firms with similar product offerings (e.g., two restaurants offering the same cuisine with comparable food quality and prices) compete for customers is another fruitful direction for future research.

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Appendix

A. Sample Instructions for Unordered Treatment

Instructions

General. The purpose of this session is to study how people make decisions in a particular situation. If you have any questions, feel free to raise your hand and a monitor will assist you. From now until the end of the session, please do not communicate with other participants in the room.

During the session, you will play a game from which you can earn 'Experimental Currency Units (ECUs)'. The ECUs you earn will be converted into U.S. dollars at a rate of 3000 ECUs per $1. Upon completion of the game, you will be paid your total earnings in U.S. cash plus a $5 show-up fee.

Description of the game. You are a manager at a company that sells tickets for an upcoming event (e.g., Broadway show). You have 10 tickets to sell, in total.

There are two types of customers: *low-revenue* from whom you earn 20 ECUs, or *high-revenue* from whom you earn 200 ECUs. Each customer demands one unit.

Each round of the game consists of 600 periods. In each period, a low-revenue customer arrives with probability Pr(Low) = 0.0244, or a high-revenue customer arrives with probability Pr(High) = 0.0083, but both customers cannot arrive in the same period. This indicates that the probability of no customer arrives in a period is Pr(No customer) = 0.9673. The probability distributions for the total number of each type of customers for a round are also shown on a separate page.

The total number of high-revenue and the total number of low-revenue customers in any round are independent from each other. Also, the total number of high-revenue and the total number of low-revenue customers for any one round are independent of the demand from other rounds. Thus, a small or large demand in a round has no influence on whether demand is small or large in other rounds.

In each round of the game, when a customer arrives, you decide whether to *accept* or *reject* this demand.

- The maximum revenue that you can earn from any customer is high-revenue (i.e., 200 ECUs). Thus, you never reject a high-revenue customer as long as you have available capacity left.

- Whenever a low-revenue customer arrives, you need to decide whether to accept or reject this customer. If you accept too many low-revenue customers, you sell the units at low-revenue instead of selling them at high-revenue, thus lose 180 ECUs per unit (the difference between high-revenue and low-revenue). If you accept too few low-revenue customers, you may have unsold units at the end and forego 20 ECUs per unit (low-revenue).

Overall for a round, you need to decide when to accept or reject low-revenue customers.

Your goal is to maximize the total revenue.

Calculating revenue. For each round, computer generates random low-revenue and high-revenue demands. When a customer arrives, if you accept this demand you earn its associated revenue; if you reject it, then you simply earn 0 ECU. Assume that, overall for a round, you have accepted y units of low-revenue customers, then your revenue for this round is calculated as follows:

$$\text{Revenue} = 200 \times \min\{10 - y, \textit{High-demand}\} + 20 \times y$$

For example, suppose you accept 6 low-revenue customers for a round. This indicates that you can sell at most 4 units to high-revenue customers.

- Assume that the high-demand is 5, then your revenue for this round is:
  $200 \times \min\{4, 5\} + 20 \times 6 = 920$ ECUs.

- Now, assume that the high-demand is 2, then your revenue for this round is:
  $200 \times \min\{4, 2\} + 20 \times 6 = 520$ ECUs.

Note that when the demand for a high-revenue customer turns out to be lower than the remaining capacity not used for low-revenue customers, you lose revenue opportunities for sale of this capacity.

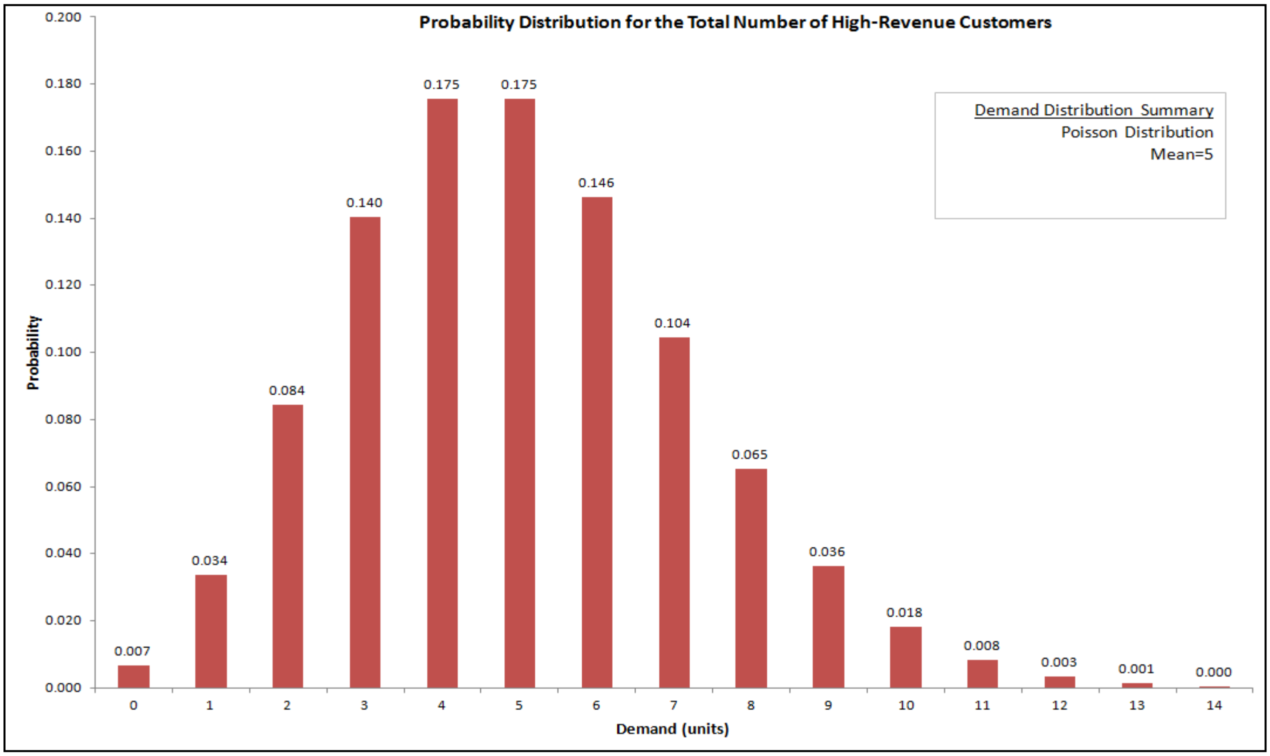
Information to help you in your decision. A pen and blank sheet of paper have been provided for any calculations or notes you might wish to make.

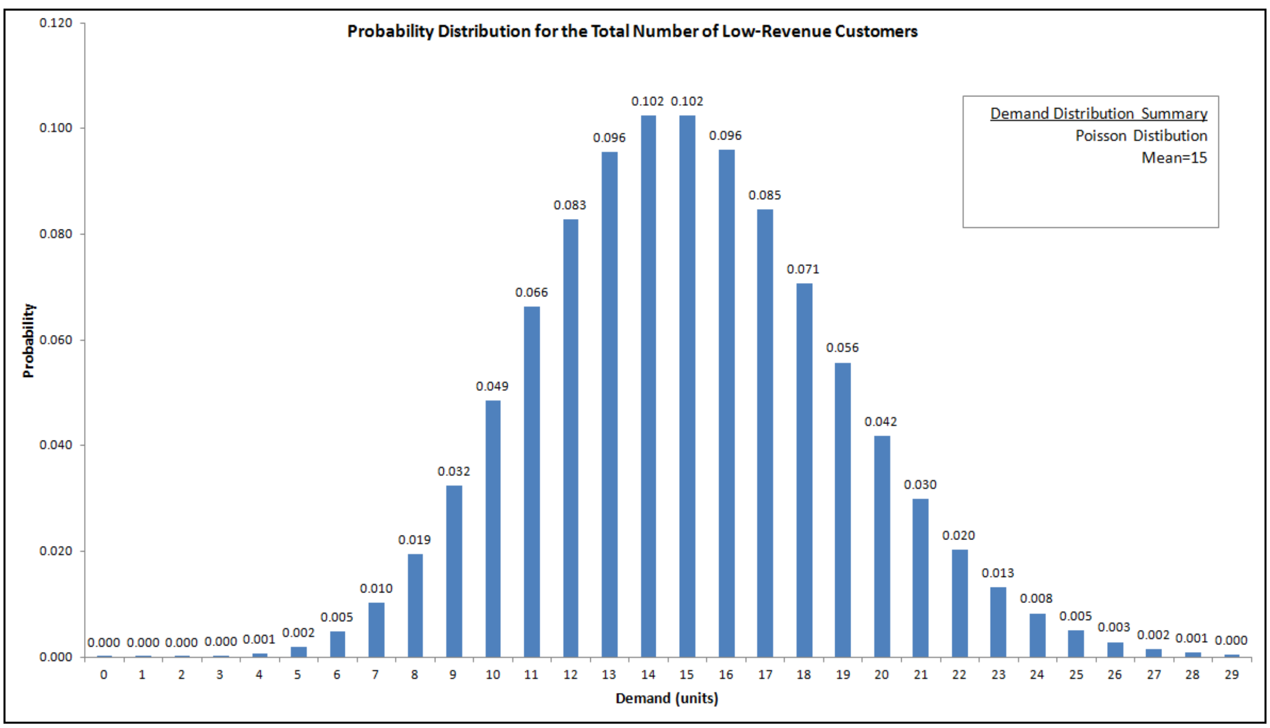
In a period, whenever a customer arrives, you will be provided information on (i) the remaining periods in this round, (ii) how much capacity you have allocated to each of the high-revenue and low-revenue customers thus far in current round, and (iii) your current revenue in this round. At the end of each round, the computer will display the history of play (how much capacity you allocated to each of the customers, the realized high-revenue and low-revenue demands, the unsold capacity you had, and your resulting total revenue) for each round.

Number of rounds. The game lasts for 40 rounds.

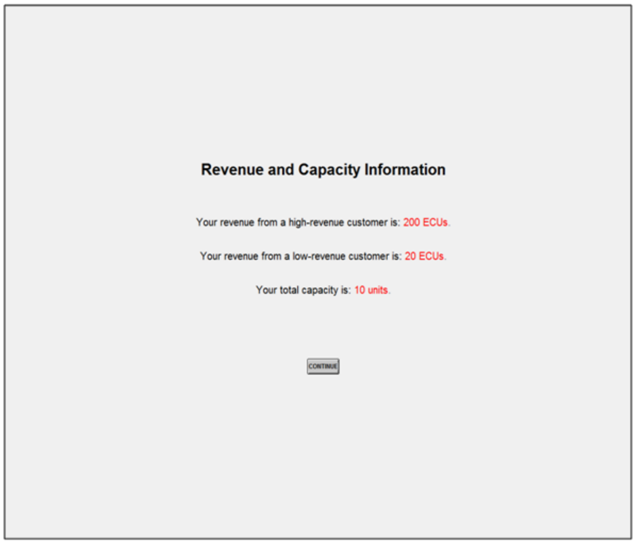
Consent Forms. If you wish to participate in this study, please read the accompanying consent form prior to beginning the game.

Demand Distribution Information

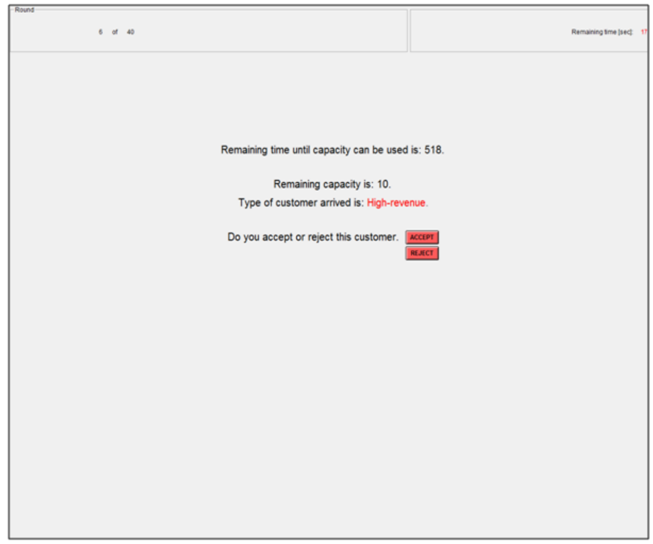




B. Sample Screenshots for Unordered Treatment



(a)



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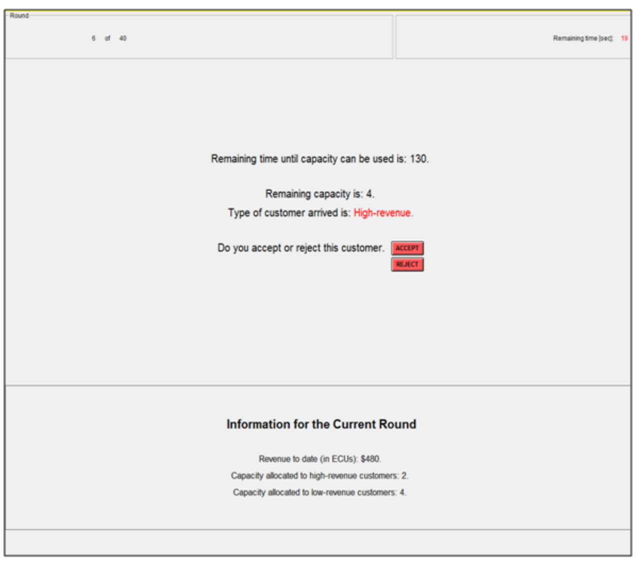

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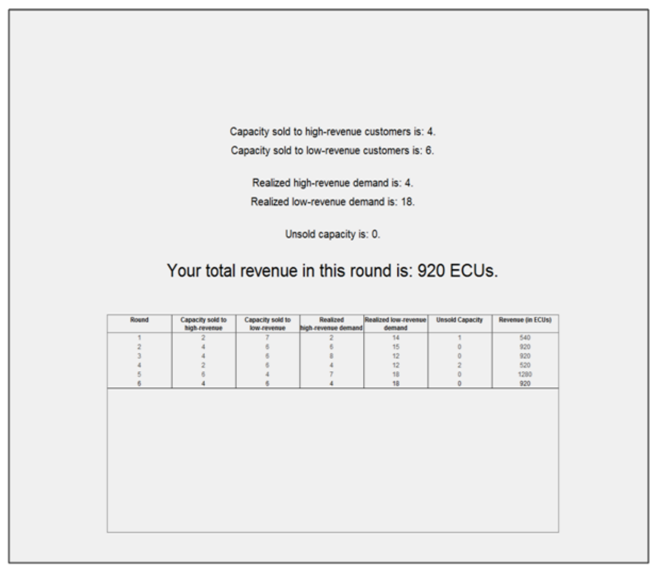

Round	Capacity sold to high-revenue	Capacity sold to low-revenue	Realized high-revenue demand	Realized low-revenue demand	Unsold Capacity	Revenue (in ECUs)
1	2	7	2	14	1	540
2	4	6	6	15	0	920
3	4	6	8	12	0	920
4	2	6	4	12	2	520
5	6	4	7	18	0	1280
6	4	6	4	18	0	920

(d)