Behavioral Anomalies in Consumer Wait-or-Buy Decisions and Their Implications for Markdown Management

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A decision to buy an item at a regular price or wait for a possible markdown involves a multi-dimensional trade-off between the value of the item, the delay in getting an item, the likelihood of getting it and the magnitude of the price discount. Such trade-offs are prone to behavioral anomalies/regularities by which human decision makers deviate from the discounted expected utility model, the benchmark adopted in the existing markdown management literature. In this paper we build an axiomatic framework that accounts for three well-known anomalies, and produces a parsimonious generalization of discounted expected utility. We consider a Stackelberg-Nash game between a firm that decides the markdown discount and a continuum of consumers who decide to wait or buy, anticipating other consumers' decisions and the resultant likelihood of product availability that emerge in the equilibrium. We solve the markdown management problem analytically, and contrast the results of our model to those under the discounted expected utility. Finally, we elicit the realistic values of model parameters by means of a laboratory experiment. We show that accounting for the behavioral anomalies results in substantially larger markdowns than the current literature suggests, and leads to noticeable revenue gains.

Key words: Retail Operations, Markdown Management, Price and Probability Tradeoff, Behavioral Anomalies, Behavioral Operations

History: This version: November 1, 2014

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1. Introduction

This paper deals with a fundamental decision that consumers in developed economies make on a regular basis: buy an item now at the tag price p, or wait until time t when the product will be marked down at a price $p(1-d)$, but may only be available with probability q. Understanding how consumers make such *wait-or-buy decisions* is crucial for retailers to properly optimize markdowns. Markdowns in turn are critically important for retailers as they correspond to approximately a third of unit sales and a fifth of dollar sales (Smith and Achabal 1998, Agrawal and Smith 2009).

Our starting point is the observation that the wait-or-buy decision is a four-dimensional trade-off between the value of the item, the magnitude of the price discount, the likelihood of getting the item, and the delay in getting it. The existing markdown management literature assumes that consumers use the discounted expected utility (DEU for short) model to resolve these trade-offs. As we discuss in §2, trade-offs in each of these dimensions are prone to behavioral anomalies by which human decision makers deviate from DEU. The reason the DEU model fails to explain these anomalies is because it treats each dimension in a linear and separate way, whereas people view them in a non-linear and interdependent way. For instance, individuals' sensitivity to risk of not obtaining an item depends on the time delay and the magnitude of the discount, sensitivity to time delay depends on the discount and risk, and so on.

We capture this interdependency through the notion of *psychological distance* between the prospects of "buy now for sure" (zero distance) and "perhaps buy later" (both time and risk distance). Using this notion in §3 we develop a new preference model of consumer wait-or-buy choice that accounts for three sets of well-known behavioral anomalies. These anomalies are: the common ratio effect describing non-linearity in sensitivity to risk; the common difference effect (a.k.a., hyperbolic discounting), describing the non-linearity in sensitivity to time; and subendurance, describing how sensitivity to time depends on the magnitude of the payoff. Previous research considered models that account for these individual anomalies, but there appears to be no model that accounts for these anomalies *simultaneously*, which is necessary to describe a wait-or-buy decision because is combines risk, time and payoff. Rather than simply guessing a model that would do such a job – and this is one of the key contributions of our paper – we set preference conditions (axioms) that build on these anomalies. Based on these axioms we then *derive* the new model that captures the interdependencies between individuals' perceptions of time, risk and payoffs as are manifested through these anomalies. The result is a parsimonious generalization of the DEU model.

We then use our new model to optimize retailer's markdown discount, and compare it to the one optimal under the DEU model of the current literature. To do so we consider a Stackelberg-Nash game between the firm and a continuum of consumers. As described in §4, consumers (Stackelberg followers), given the discount, anticipate other consumers' decisions and the resultant probability of product availability, all of which are endogenously determined in the Nash equilibrium. Anticipating this equilibrium, in §5 the retailer (Stackelberg leader) solves the markdown optimization problem. Through a combination of analytical and numerical results we show that, compared with the discounted expected utility benchmark, our behavioral model leads to larger optimal markdowns, *and* larger revenues. Smith and Achabal (1998) and Agrawal and Smith (2009) argue that retailers should offer larger markdowns based on the dependency between the demand and remaining available inventory, and Özer and Zheng (2014) argue that dynamic pricing is even more valuable than previously thought because of the non-pecuniary behavioral[1] factors such as consumers' regret and misperceptions of product availability. We reach the same conclusion based on the consumers' psychological perceptions of time, risk and price discount.

To understand this result recall that under DEU the optimal markdown is selected to balance the marginal revenue from selling more units at the markdown price with the marginal cost of diverting consumers from buying at the tag price. The behavioral anomalies we study affect this balance in two ways. First, subendurance implies that consumers are less patient for small markdowns, and more patient for large ones. At the markdown level optimal for DEU, the former effect dominates, thus a retailer can exploit this impatience and offer larger markdowns without sacrificing sales at the tag price. Second, the non-linearities in risk and time perception in our model imply that consumers are more sensitive to psychological distance than the DEU model assumes when distances are small, but are less sensitive when distances are large. Increasing the markdown increases demand, which increases product availability risk and therefore the psychological distance. Thus, consumers who at the DEU-optimal markdown were 'buying now' (i.e., had a zero distance) are very sensitive to increased distance; consequently they continue to buy now. Likewise, those who were waiting under the DEU are continuing to wait as they are less sensitive to the increased distance because waiting implies a positive psychological distance to begin with. The subendurance and non-linearity effects complement each other and allow a retailer to offer larger markdowns and gain additional revenue.

To assess realistic values of markdown increases and revenue gains in §6 we elicit model parameters through a laboratory experiment. To ensure the quality of the data we used binary questions and choice-lists (Holt and Laury 2002), in which participants face a battery of wait or buy choices under different price discount and risk scenarios. We observe that subjects' responses are remarkably consistent and reveal clear indifference points at which subjects switch from buying to waiting. The existence of such indifference points implies that the observed decisions are non-trivially correlated. We therefore use indifference points (and not the raw choice data) to fit our model. Further, we use the least absolute deviation (quantile) regression (Koenke and Hallock 2001) to take care of the censored error structure. To make sure individuals are properly motivated we adopt a refined version of the randomized incentive scheme (RIS), called PrInce. In RIS, after the experiment, one of the choices faced by a subject is randomly selected and played for real; in PrInce, one choice is

[1] For broader discussion of behavioral issues in pricing see Özer and Zheng (2012); Chapters 2.1 and 3.1.2 are particularly relevant for our study.

randomly pre-selected and given to a participant in a sealed envelope before the experiment; this has been shown to improve the quality of eliciation (Johnson et al. 2014).

After estimating the parameters, we use these to solve our game: calculate the associated consumer wait-or-buy equilibrium and anticipating it, optimize the retailer's markdown discount. The resulting optimal discount is 5-10% larger, and the revenue is 1-1.5% larger, than what would be optimal if the retailer were to assume that consumers behave according to the discounted expected utility model. Note that a typical retailer operates with a net margin of approximately 3%[2]; hence our new, behaviorally-inspired model has a potential to significantly increase retailers' profitability.

Our paper contributes to two bodies of behavioral operations literature. First, it is among the few recent studies that enrich dynamic pricing models with various behavioral phenomena. These include: regret in Nasiry and Popescu (2012), anecdotal reasoning in Huang and Liu (2014), uncertain product value in Swinney (2011), stockpiling and inertia in Su (2010), Su (2009), probability mis-perception in Özer and Zheng (2014), as well as the "standard" loss-aversion, risk-aversion and reference dependence, e.g., Popescu and Wu (2007), Liu and van Ryzin (2008), Tereyagoglu et al. (2014). The innovativeness of our approach is that we study a more basic question of *how* consumers decide to wait or buy, and from that derive the resultant behavioral model, in addition to studying the impact of such a behavior. Our paper also adds to the empirical literature on strategic consumers, e.g., Mak et al. (2014), Kim and Dasu (2014), Li et al. (2014), Osadchiy and Bendoly (2010). But rather than using the experiment to substantiate the model, our experiment only parameterizes the model that is developed based on the existing evidence of behavioral anomalies.

2. The DEU Model and the Behavioral Anomalies it Fails to Explain

The existing literature on markdown management (e.g., Besanko and Winston 1990, Aviv and Pazgal 2008, Liu and van Ryzin 2008, Zhang and Cooper 2008) uses the discounted expected utility (DEU) model to solve the wait-or-buy problem. Let u denote the willingness to pay in the 'buy now' case, which we call *the benefit of consumption*. By default, 'buy now' ensures the purchase. According to DEU, to 'opt out' yields 0 utility, 'buy now' yields $u-p$, and 'wait' yields

$$U(p,d,q,t) = [u - p(1-d)]\; q\; e^{-rt}. \tag{1}$$

Here, $r > 0$ denotes the time discount rate. According to DEU, the consumer will opt-out if $u < p(1-d)$, wait if $p(1-d) \le u < H^{DEU}$, and buy now if $u \ge H^{DEU}$, where

$$H^{DEU} = p \cdot \frac{1-(1-d)qe^{-rt}}{1-qe^{-rt}}.$$

[2] http://pages.stern.nyu.edu/∼adamodar/New_Home_Page/datafile/margin.html

The solution is intuitive: if u is high, the penalty for availability risk and/or discounting is higher, prompting the consumer to buy now. Moreover, the higher the price discount, the more attractive is to wait, but this attractiveness is dampened if either t is large or q is small. Finally, $H^{DEU} \geq p$ because for consumers with $p(1-d) \leq u < p$ the only profitable option is to wait.

We believe that DEU is directionally correct – individuals like price discounts and dislike availability risk and delay. But DEU fails to account for three behaviorally important effects.

The first is the *common ratio effect* in risk preferences, by which the effect of the probability q is not linear in the mind of the consumer. A 20% change in the probability of the product being available from 100% to 80% has a much higher relative impact than the same 20% change from 50% to 40%. People seem to be less sensitive to probability ratios when probabilities become small. The following example may clarify. Consider two stores. The product is offered in store A with no discount and 100% availability, and in store B with a discount of 30% and availability risk 80%. According to DEU, if the consumer is indifferent between A and B, then he should also be indifferent between C and D, where 'C: no discount and 50% availability risk' and 'D: 30% discount and availability risk of 40%'. Intuitively, and experimentally, many subjects prefer D to C.

The second is the *common difference effect* in time preference (a.k.a., hyperbolic discounting). Changing the delay from 0 (no delay) to 7 days has a higher relative impact that a change from 21 days to 28 days. People seem to be less sensitive to delays when consequences are far into the future. Consider two options. Option A is to buy the product now at no price discount, and option B is to buy the product one weeks later at a price discount of 20%. Assume the product is available for sure in all options. According to DEU, if the consumer is indifferent between A and B, then he should also be indifferent between C and D, where 'C: buy in three weeks at no discount' and 'D: buy in four weeks at a 20% discount'. Intuitively, and experimentally, people prefer D to C.

The third effect is known as the *magnitude effect* in time preferences. The benchmark model assumes that the time discount rate, r, is a fixed parameter. There is an abundant evidence that the degree of impatience is higher for small consequences than it is for large consequences (Thaler 1981, Frederick et al. 2002). Baucells and Heukamp (2012) propose a preference condition, called *subendurance*, that captures the following preference. Most individuals seem to prefer 'A: a gain of \$20 now with 50% chance' to 'B: a gain of \$20 in six months for sure', but when considering the same decision with higher stakes, namely 'C: a gain of \$1,000 now with 50% to chance' or 'D: a gain of \$1,000 in six months for sure' they reverse their choice and prefer D to C.

Table 1 shows experimental evidence for these three anomalies. Pattern 1-2 replicates the common ratio effect, a violation of proportionality. Pattern 3-4 reproduces the common difference effect, a violation of stationarity. These two effects are at the core of a considerable literature on non-linear probability weighting (Allais 1953, Wakker 2010) and hyperbolic discounting (Laibson 1997, O'Donoghue and Rabin 1999, DellaVigna and Malmendier 2004). Choices 5 and 6 reflect the pattern of subendurance.

	Prospect A	v.	Prospect B	Response	N
1.	**(9 €, for sure, now)**	v.	(12 €, with 80%, now)	**58%** v. 42%	142
2.	(9 €, with 10%, now)	v.	**(12 €, with 8%, now)**	22% v. **78%**	65
3.	**(100 fl, for sure, now)**	v.	(110 fl, for sure, 4 weeks)	**82%** v. 18%	60
4.	(100 fl, for sure, 26 weeks)	v.	**(110 fl, for sure, 30 weeks)**	37% v. **63%**	60
5.	(5 €, for sure, 1 month)	v.	**(5 €, with 90%, now)**	43% v. **57%**	79
6.	**(100 €, for sure, 1 month)**	v.	(100 €, with 90%, now)	**81%** v. 19%	79

Table 1 **Rows 1-2 are taken from Baucells and Heukamp (2010, Table 1). Rows 3-4 are taken from Keren and Roelofsma (1995, Table 1) (1 fl or Dutch Gulden in 1995 = $0.6). Rows 5-6 from Baucells et al. (2009).**

The DEU benchmark is highly incompatible with these patterns. Indeed, pattern 1-2 is incompatible with linear probability weighting, and pattern 3-4 is incompatible with exponential discounting (Baucells and Heukamp 2012, Proposition 1). Pattern 5-6 requires that time discounting be affected by the outcome dimension.

Our goal is to propose a modification of DEU that better approximates how individuals feel about the trade-offs between price, price discounts, probabilities, and delays (i.e., all the patterns in Table 1). In what follows, rather than guessing a utility model, we will propose preference conditions characterizing such a model.

Our model rests on the axiomatic preference framework of Baucells and Heukamp (2012). Same as they do, we assume that the risk and time distances, $\ln 1/q$ and t, are substitutes; that their "exchange rate" r may depend on the outcome; and that individuals exhibit diminishing sensitivity to distance, i.e., the probability and delay penalty is a concave function of $\ln 1/q + r \cdot t$. This captures the certainty/immediacy effect and the possibility effect (a.k.a., the long-shot bias) (Kahneman and Tversky 1979). That is, consumers are disproportionally sensitive to a small change from full and immediate availability to partial availability or small delay. On the flip side, consumers are less sensitive to additional delays, or increases in availability risk, if the prospect is in the future or not certain to begin with.

In our setup, the outcome has two dimensions: price and price discount. We will assume that it is the price discount that drives the subendurance effect. Simply put, consumers will be more patient if the price discount is high, which will imply that r is a decreasing function of d. This assumption is consistent with Kahneman and Tversky (2000)'s observation that individuals are willing to travel 10 minutes to grab a 33% price discount on a calculator that costs \$15, but not willing to travel the same 10 minutes to grab a 5% price discount on a jacket that costs \$100. That the dollar discount is the same, \$5, shows that what drives the customer acceptance of the delay of 10 minutes depends on the price discount percentage, not the dollar value. The assumption is psychologically plausible, as the price discount d is a number that is comparable across purchases.

3. Price Discount, Probability, and Time Tradeoff

In this section we propose a set of axioms that characterize a preference relation capable of explaining the behavioral anomalies described above. Let τ denote the current calendar date. At time τ, the consumer exhibits preferences between pairs in $\mathcal{X}_\tau = [0,\infty) \times [0,1] \times [0,1] \times [\tau,\infty]$, where a typical element will be written as $x = (p_x, d_x, q_x, t_x) \in \mathcal{X}$. Here, p_x represents the tag price, d_x the price discount, q_x the probability of the good being available, and $t_x \geq \tau$ is the purchase date. Each consumer desires one item. The benefit of consumption does not depend on τ. Note that we allow for $t_x = \infty$ (never). The availability of the item is revealed at time t_x.

A word on notation. We write (d, x_{-d}) or (q, t, x_{-qt}) to denote the vectors (p_x, d, q_x, t_x) or (p_x, d_x, q, t), respectively. Throughout, 'decreasing' implies 'non-increasing', and we use 'strictly decreasing' otherwise. Same holds for 'increasing' or 'concave'. Throughout, the time and risk distance of x is t_x and $\ln 1/q_x$, respectively. Finally, we let $\mathbf{0} = (0,0,0,0)$.

3.1 Axioms

Let $\succeq_\tau$ denote a preference ordering over pairs in $\mathcal{X}_\tau$ as expressed by a consumer from the point of view of the current calendar time τ. The first four and the last axiom are technical in nature. Axioms 5 to 7 capture the behavioral anomalies in preferences. Our first axiom will guarantee the existence of a continuous function, $V_\tau(p,d,q,t)$, that represents such preferences.

A1. For each $\tau \geq 0$, $\succeq_\tau$ is a *complete and continuous ordering* over $\mathcal{X}_\tau$.

The next axiom states that preferences are not a function of calendar time, but a function of time relative to τ. It translates reference-dependence (outcomes are not evaluated in absolute, but relative to a reference point) into the time dimension.

A2. *Time invariance.* $\forall x, y \in X$, $0 \leq \tau \leq t_x, t_y$, and $\Delta \geq 0$,

$$x \sim_\tau y \quad \text{if and only if} \quad (t_x + \Delta, x_{-t}) \sim_{\tau+\Delta} (t_y + \Delta, y_{-t}).$$

Time invariance implies that $V_\tau(p,d,q,t) = V_0(p,d,q,t-\tau)$. Hence, specifying the preferences from the viewpoint of $\tau = 0$ automatically determines the preferences from all time viewpoints. Henceforth, when we omit the subscript τ from $\mathcal{X}$, V, and $\succeq$ it means that $\tau = 0$.

Next, we impose monotonicity and solvability conditions. Null purchases, those having $q=0$ or $t=\infty$ are interpreted as no purchases and are deemed indifferent. The directional effects of price, price discounts, time, and probability are the same as in DEU. Finally, while the item is desirable for free, there is a finite price one is willing to pay.

A3. *Monotonicity and Solvability.* Let $\mathcal{X}^0 = \{x \in \mathcal{X} :\ q_x = 0 \text{ or } t_x = \infty\}$. For all $x \in \mathcal{X}$,

A3.0 if $x \in \mathcal{X}^0$, then $x \sim \mathbf{0}$.

A3.p let $p < p_x$. If $x \notin \mathcal{X}^0$, then $(p, x_{-p}) \succ x$.

A3.d let $d > d_x$. If $x \succ \mathbf{0}$, then $(d, x_{-d}) \succ x$.

A3.q let $q > q_x$. If $x \succ \mathbf{0}$, then $(q, x_{-q}) \succ x$; and if $x \prec \mathbf{0}$, then $(q, x_{-q}) \prec x$.

A3.t let $t < t_x$. If $x \succ \mathbf{0}$, then $(t, x_{-t}) \succ x$; and if $x \prec \mathbf{0}$, then $(t, x_{-t}) \prec x$.

A3.u there exist a $u \in (0, \infty)$ such that $(u, 0, 1, 0) \sim \mathbf{0}$.

Because A3.u is imposed for immediate sure purchases with no price discount, u does not depend on the dimensions of x, but only on the item itself. By time invariance, u does not change with the passage of calendar time.

Next, we assume that, for immediate purchases, price and price discount are rationally encoded in a way that only the effective price matters.[3]

A4. *Effective Price Condition.* For all $x, y \in \mathcal{X}$ such that $q_x = q_y$ and $t_x = t_y = 0$,

$$x \succeq y \text{ if and only if } p_x(1-d_x) \leq p_y(1-d_y).$$

The next condition links risk and time preferences (Baucells and Heukamp 2012).[4] The condition captures the psychologically intuitive notion that "time is intrinsically uncertain". Intuitively, if a delay of $\Delta = 1$ month is exchangeable with a probability factor of $\theta = 80\%$, then a delay of $\Delta = 2$ months is exchangeable with a probability factor of $\theta^2 = 64\%$, and this exchange holds independently of the base level of time and probability. For any delay $\Delta > 0$, one can find a reduction in probability $\theta < 1$ (without the delay) that offsets the effect of Δ. The condition states is that once this trade-off is established at some probability and time base levels, it holds for all probability and time base levels as well as all price levels. The condition does not extend to different price discounts because the probability and time trade-off may depend on d_x.

[3] The condition precludes framing effects, i.e., changes in preferences associated with increasing the tag price and the price discount simultaneously while keeping the same effective price. The condition is imposed only on immediate purchases and may not hold for delayed purchases.

[4] Time preferences are often stated by means of a trade-off between a delay and an improvement in some outcome dimension. The outcome dimensions (e.g., income, consumption, or a multi-attribute description of a consequence), however, varies from problem to problem. Because probability and time are dimensions of almost any decision, the a trade-off between a delay and an improvement in probability is a more portable frame.

A5. *Probability and time trade-off.* $\forall x \in \mathcal{X}$, $p \geq 0$, $\theta, q \in [0,1]$ and $\Delta, t \in [0, \infty]$,

$$(p_x, d_x, q_x, t_x + \Delta) \sim (p_x, d_x, q_x\theta, t_x) \text{ if and only if } (p, d_x, q, t + \Delta) \sim (p, d_x, q\theta, t).$$

The indifference in A5 allows us to define the exchange rate between probability and time,

$$r(d) = \frac{1}{\Delta} \ln \frac{1}{\theta}.$$

By A5, $r(d)$ does not depend on q, t, and p, but may depend on the price discount (not so in the case of DEU). The exchange rate, r, will be called the probability discount rate.

A5 is compatible with, but logically independent from, the three preference patterns exhibited in Table 1. To explain pattern 5-6 it is necessary to let the probability discount rate depend on the outcome dimension. In order to reduce degrees of freedom, and as explained in §2, we assume that what drives probabilistic patience is the price discount only.

A6. *Price Discount Subendurance.* $\forall x \in \mathcal{X}$, $\theta \in [0,1)$, $\Delta \in [0, \infty]$, and $d > d_x$,

$$\text{if } x \succ \mathbf{0} \text{ and } (t_x + \Delta, x_{-t}) \sim (q_x\theta, x_{-q}), \text{ then } (p_x, d, q_x, t_x + \Delta) \succeq (p_x, d, q_x\theta, t_x).$$
$$\text{if } x \prec \mathbf{0} \text{ and } (t_x + \Delta, x_{-t}) \sim (q_x\theta, x_{-q}), \text{ then } (p_x, d, q_x, t_x + \Delta) \preceq (p_x, d, q_x\theta, t_x).$$

A6 implies that individuals become more patient for higher price discounts, i.e., $r(d)$ is decreasing in d. This also ensures that a higher price discount makes an attractive product even more attractive.

The following axiom considers patterns 1-2 and 3-4. The first condition captures the common ratio effect (a reduction in probabilities makes the prospect with the better outcome more attractive). The second condition captures the common difference effect (a delay makes the prospect with the better outcome more attractive). Both conditions reflect a loss of sensitivity when risk distance and time distance are increased.

A7. *Sub-proportionality and Sub-stationarity.*

A7.p Let $x, y \in \mathcal{X}$ with $q_x \leq q_y$, $t_x = t_y$. For all $\theta \in [0,1]$,

$$\text{if } x \sim y \succ \mathbf{0} \text{ then } (\theta q_y, y_{-q}) \preceq (\theta q_x, x_{-q}); \text{ and}$$
$$\text{if } x \sim y \prec \mathbf{0} \text{ then } (\theta q_y, y_{-q}) \succeq (\theta q_x, x_{-q}).$$

A7.t Let $x, y \in \mathcal{X}$ with $t_x \geq t_y$, $d_x \geq d_y$, and $q_x = q_y$. For all $\Delta \geq 0$,

$$\text{if } x \sim y \succ \mathbf{0} \text{ then } (t_y + \Delta, y_{-t}) \preceq (t_x + \Delta, x_{-t}); \text{ and}$$
$$\text{if } x \sim y \prec \mathbf{0} \text{ then } (t_y + \Delta, y_{-t}) \succeq (t_x + \Delta, x_{-t}).$$

If we strengthen A7 by implying indifference, then we obtain *Proportionality* and *Stationarity*, respectively. Proportionality and stationarity are the essence of DEU, but have been disconfirmed numerous times in experiments. The 'sub' conditions are behaviorally more plausible.

Our final axiom produces a simple structure by assuming separability between the price and the probability dimensions, but only for prospects received now at no discount.[5]

A8. *Restricted Probability-Price separability.* For all $x \in X$ with $t_x = 0$ and $d_x = 0$, $p, p', p'' \geq 0$, $q, q', q'' \in [0,1]$, if three of the following indifferences holds, the fourth one holds as well.

$$(p, q', x_{-pq}) \sim (p', q, x_{-pq}) \qquad (p', q', x_{-pq}) \sim (p'', q, x_{-pq})$$
$$(p, q'', x_{-pq}) \sim (p', q', x_{-pq}) \qquad (p', q'', x_{-pq}) \sim (p'', q', x_{-pq})$$

3.2 Representation

A1-A8 is our complete list of axioms. The representation that follows [$x \succeq_\tau y$ iff $V_\tau(p_x, d_x, q_x, t_x) \geq V_\tau(p_y, d_y, q_y, t_y)$] will involve three continuous functions: a *value function*, $v : \mathbb{R} \to \mathbb{R}_+$, strictly increasing and with $v(0) = 0$; a *psychological distance function*, $s : \mathbb{R}_+ \to \mathbb{R}_+$, strictly increasing with $s(0) = 0$, $s(1) = 1$, and $s(\infty) = \infty$; and a *probability discount function*, $r : [0,1] \to (0, \infty)$.

Proposition 1. *$\succeq_\tau$ on $\mathcal{X}_\tau$ satisfies A1-A8 if and only if, for some value function, v, two **concave** psychological distance functions, s^+ and s^-, and some **decreasing and bounded** probability discount function, r,*

$$V_\tau(p, d, q, t) = \begin{cases} v(u - p(1-d)) \cdot e^{-s^+(\sigma)}, & u - p(1-d) \geq 0 \\ v(u - p(1-d)) \cdot e^{-s^-(\sigma)}, & u - p(1-d) < 0, \end{cases} \tag{2}$$

*where $\sigma = \ln 1/q + r(d)(t - \tau)$ is the **psychological distance** of the prospect $(p, d, q, t) \in \mathcal{X}_\tau$.*

All proofs are presented in the Appendix. We call (2) the *dPTT model*, for (price) discount-probability-time trade-off. The model builds on the three behavioral patterns of Table 1 via axioms A6 and A7. We remark several properties of the representation.

1. dPTT collapses into DEU if v, s^+, and s^- are the identity function and $r(d)$ is set constant.
2. For immediate purchases, (2) agrees with a prospect theory like formulation in which v is a value function and $w(q) = e^{-s(-\ln q)}$ is a sub-proportional probability weighting function.
3. For future purchases with no availability risk, (2) agrees with a hyperbolic discounting model in which $f(t) = e^{-s(t)}$ is a sub-stationary time discount function.

[5] A8 is known as the hexagon condition, and it is a specialization of the *corresponding trade-off condition* (Keeney and Raiffa 1976, Theorem 3.2) for the case of two attributes. Karni and Safra (1998) shows that the hexagon condition is equivalent to the Thomsen condition.

4. The list of axioms could be shorter. Intuitively, if individuals exhibit diminishing sensitivity to risk distance (A7.p), and risk and time distance are substitutes (A5), then they should exhibit diminishing sensitivity to time distance (A7.t). Also, monotonicity with respect to price discounts (A3.d) follows from monotonicity with respect to price and the effective price condition.

   PROPOSITION 2. *A3.d and A7.t are implied by the rest of conditions in A1-A7.*

5. dPTT possesses a minimum of time consistency. For instance, null purchases will be deemed indifferent both at $\tau = 0$ and at any subsequent time $\tau > 0$. Moreover, with the passage of time, favorable deals will remain favorable; and unfavorable deal will remain unfavorable. Importantly, if a product is attractive today, but the consumer decides to wait, then the product will remain attractive in the future upon learning that the product is available.

   PROPOSITION 3. *Assume A1-A4 and let* $\mathbf{0}_\tau = (0,0,0,\tau)$*.*

   - *If* $x \sim \mathbf{0}$*, then* $x \sim_\tau \mathbf{0}_\tau$*; if* $x \succ_0 \mathbf{0}$*, then* $x \succ_\tau \mathbf{0}_\tau$*; and if* $x \prec_0 \mathbf{0}$*, then* $x \prec_\tau \mathbf{0}_\tau$*,* $\tau \le t_x$*.*
   - *If* $(p,d',q,t) \succeq_0 (p,d,1,0) \succ_0 \mathbf{0}$ *and* $d' \ge d$*, then* $(p,d',1,t) \succ_t \mathbf{0}_t$*.*

6. The term $s(\ln 1/q + r(d)t)$ implies that risk and time distance are substitutes, and that individuals exhibit diminishing sensitivity to distance (of either type). Thus, adding distance (of either type) reduces sensitivity to additional distance (of either type). This observation affects our markdown problem as follows: because the option of waiting always exhibits time distance, individuals will not be very sensitive to probability reductions, even for q close to one.[6]

3.3 Parametric dPTT model

Because the consumer can opt-out from undesirable prospects, there is no point in modeling how much negative utility they produce. Henceforth, we restrict attention to desirable prospects, $u \ge p(1-d)$, and omit the superscript '+' from s. To gain tractability and further proceed with studying the markdown management problem we adopt the following parametrized form:

- For v, we adopt $v(u - p(1-d)) = u - p(1-d)$. This linear form is widely used in management science and economics models.

[6] That risk and time distance are substitutes yields two nontrivial predictions, namely, that the common ratio effect can be reproduced by adding a common delay to both options; and that the common difference effect can be reproduced using a common probability reduction for both options. Intuitively, both manipulations increase the distance $\sigma = \ln 1/q + r(d)t$, and because $s(\sigma)$ is concave, induce loss of sensitivity with respect to either probability or time and make the payoff dimension more salient. In particular, adding a common delay of 3 months to choice 1 in Table 1 would produce a reversal. Indeed, Baucells and Heukamp (2010, Table 1) show that only 43% of subjects prefer (9 €, for sure, 3 months) to (12 €, with 80%, 3 months). Similarly, adding a common reduction of probability to choice 3 in Table 1 would produce a reversal. Indeed, Keren and Roelofsma (1995, Table 1) show that only 39% of subjects prefer (100 fl, with 50%, now) to (110 fl, with 50%, 4 weeks).

- For s, we adopt $s(\sigma) = \sigma^\beta$, $0 < \beta \le 1$. This power form is associated with Prelec (1998)'s probability weighting function, $w(q) = e^{-(-\ln q)^\beta}$ and Ebert and Prelec (2007)'s time discount functions, $f(t) = e^{-(t)^\beta}$. These specifications of w and f fit the experimental data quite well (Ebert and Prelec 2007, Booij et al. 2010) in the risk-only and time-only domains, respectively.
- For $r(d)$, which is new in the literature, we propose the bounded and decreasing form

$$r(d) = \rho\ e^{\mu(d_0 - d)}, \quad \rho > 0,\ \mu \ge 0,\ d_0 \in [0,1].$$

The parameter μ captures subendurance, and the higher the μ, the more impatience with respect to small price discounts. The parameter d_0 represents the reference discount around which subendurance emerges. That is, the sub-endurant dPTT consumers (with $\mu > 0$) are less patient for discounts less than d_0 and more patient for discounts over d_0. The parameter ρ represents the "baseline" time discount rate without subendurance: i.e., that of a DEU decision maker (with $\mu = 0$), or equivalently, that of a dPTT decision maker facing a price discount of d_0.

The resulting model, named the parametric *dPTT model*, is given by:

$$V_\tau(p,d,q,t) = [u - p(1-d)] \cdot \exp\left\{ -\left(\ln\frac{1}{q} + \rho\ e^{\mu(d_0-d)}\ (t-\tau) \right)^\beta \right\}, \tag{3}$$

with $u \ge p(1-d)$, $\beta \in (0,1]$, $\rho > 0$, $d_0 \in [0,1]$, and $\mu \ge 0$. Setting $\beta = 1$ and $\mu = 0$ yields the DEU model. Values of β less than one will induce diminishing sensitivity to risk and time distance; and values of $\mu > 0$ will induce more patience when price discounts increase.

4. Wait or Buy Decisions Under the dPTT Model

We turn our attention to a market environment in which numerous consumers with dPTT preferences best-respond to a selling mechanism designed by the retailer.

4.1 The Selling Mechanism

The game involves one retailer (seller) and a continuum of consumers with a total mass of $\lambda > 0$. Consumers exhibit identical dPTT parameters, and differ only on the benefit of consumption u. The value of u is a private information drawn independently from a distribution with cdf $F(u)$. We assume that F is continuous with support $[0, \bar{u}]$, $\bar{u} > p$. Without loss of generality, we normalize the economy to $\bar{u} = 1$. Both λ and F are common knowledge. Throughout, $\bar{F}$ denotes $1 - F$ and $U[0,1]$ denotes a uniform distribution.

The retailer is endowed with an initial inventory Q of a homogeneous, perishable and infinitely divisible product that cannot be replenished and needs to be depleted over a two period selling season. Without loss of generality, we let time 0 be period 1 and some exogenously given $t > 0$

be the 'markdown' period 2. At time 0 (the beginning of period 1) the product is priced at the (exogenously given) tag price $p \in [0,1]$, and the retailer's markdown management problem is to decide on the discount percentage $d \in [0,1]$ to be applied to all units of unsold inventory at time t, i.e., in period 2. That is, the retailer commits to d in period 1, a typical assumption in dynamic pricing literature, e.g., Aviv and Pazgal (2008), Liu and van Ryzin (2008).

At the beginning of period 1 each consumer observes (Q,p,d) and chooses to either 'opt-out', 'wait' until time t and buy at price $p(1-d)$ but face availability risk, or 'buy now' at price p. All consumers act simultaneously and do not observe each other's choices – a Nash game. Let λ_1 and λ_2 be the mass of consumers who decide to 'buy now' and 'wait', respectively.

Clearance is modeled as an instantaneous event and calculated using a fluid model.[7] If $\lambda_1 \le Q$, then all consumers who 'buy now' do so at price p. If $\lambda_1 > Q$, then units are allocated following a lottery with each customer having equal probability of receiving an item (a proxy for random arrivals and first-come, first-served allocation). The remaining inventory is for those customers who choose to 'wait'. As before, the probability of obtaining an item in period 2 at price $p(1-d)$ is the inventory that remains available divided by the number of consumers that decided to wait. In summary, the *probability of obtaining the item* in period 1 and 2, respectively, is equal to

$$q_1 = \min\left(\frac{Q}{\lambda_1}, 1\right) \quad \text{and} \quad q_2 = \min\left(\frac{\max(Q-\lambda_1, 0)}{\lambda_2}, 1\right) \tag{4}$$

if $\lambda_1 > 0$ and $\lambda_2 > 0$; and equal to the limit of these expressions when $\lambda_1 \to 0$ and $\lambda_2 \to 0$ otherwise (i.e., equal to zero if there is no quantity available, and equal to one otherwise). Observe that customers in period 1 have priority, leading to $q_1 \ge q_2$ and $(1-q_1)q_2 = 0$ (if $q_1 < 1$, then $q_2 = 0$). Note that consumers in our model are assumed to have no mis-perception of the probability of availability; this is a qualitative difference between our paper and Özer and Zheng (2014) where consumers "cannot correctly perceive the availability of the product in the future". The behavioral anomalies therefore impact the markdown management problem through the dPTT function V.

The payoffs of a u-consumer associated with 'opt out', 'wait', and 'buy now', respectively, are

$$\begin{aligned}
&V_0(p,0,0,0) = 0,\\
&V_0(p,d,q_2,t) = [u - p(1-d)]\ e^{-s(\ln 1/q_2 + r(d)t)}, \text{ and}\\
&V_0(p,0,q_1,0) = [u - p]\ e^{-s(\ln 1/q_1)}.
\end{aligned}$$

[7] The fluid model is a limiting case of multiple probabilistic demand when the demand rate and capacity proportionally grow large (Maglaras and Meissner 2006).

All payoffs are calculated from the point of view of time 0, when the decision is made. Proposition 3 assures us that every consumer who found the item attractive in period 1 still finds it attractive in period 2. Hence, those that waited will indeed carry out the purchase.

The payoff for the retailer is given by the expected revenue (we neglect production costs which are already incurred, and the time value of money between period 1 and 2):

$$R = p \cdot \min\{\lambda_1, Q\} + p(1-d) \cdot \min\{\lambda_2, \max(Q - \lambda_1, 0)\}. \tag{5}$$

Structurally, the selling mechanism is that of a Stackelberg game, with the retailer being the leader and the consumers being the followers. As is common in the analysis of such games, we first discuss the reaction of the followers, and then in §5 discuss the problem of the leader.

4.2 Consumer's Best Response and Nash Equilibrium

Suppose a u-consumer observes (Q, p, d). How should she react? We show that the best response can be characterized by a threshold $H \in [p, 1]$.

- If $0 \le u < p(1-d)$, then 'opt-out' is a dominant strategy (buy is never profitable).
- If $p(1-d) \le u < p$, then 'wait' is a dominant strategy ('buy now' is never profitable).
- If $p \le u \le 1$, then the consumer needs to form some expectation of q_1 and q_2. For any such expectation, note that both $V_0(p, d, q_2, t)$ ('wait') and $V_0(p, 0, q_1, 0)$ ('buy now') are linear functions of u, with the 'wait' payoff having smaller slope and higher intercept. Hence, there is a unique threshold $H \in [p, 1]$ such that the consumer will wait if $p(1-d) \le u \le H$, and buy now if $H < u \le 1$.

Next, we conjecture the existence of, and restrict attention to, a symmetric equilibrium in pure strategies in which all consumers use the same threshold $H \in [p, 1]$. To characterize equilibrium we assume that all consumers use $H \in [p, 1]$; calculate any one consumer best response by means of the threshold $\mathcal{B}(H) \in [p, 1]$; and impose the equilibrium condition $\mathcal{B}(H) = H$. We denote by H^* any such solution. We begin with calculating $\mathcal{B}(H)$.

Proposition 4. *Assume all consumers use the threshold H. Then,*

$$\lambda_1 = \lambda \bar{F}(H) \quad \textit{and} \quad \lambda_2 = \lambda(F(H) - F(p(1-d))). \tag{6}$$

The best response of any one consumer is to opt out if $u < p(1-d)$, to wait if $p(1-d) \le u \le \mathcal{B}(H)$, and to buy if $u > \mathcal{B}(H)$, where

$$\mathcal{B}(H) = \min\left\{ p \cdot \frac{e^{-s(\ln 1/q_1)} - (1-d)e^{-s(\ln 1/q_2 + r(d)t)}}{e^{-s(\ln 1/q_1)} - e^{-s(\ln 1/q_2 + r(d)t)}}, 1 \right\}, \tag{7}$$

and q_1 and q_2 are as in (4). Moreover, $\mathcal{B}(H)$ is increasing in H.

Because $\mathcal{B}(H):[p,1]\to[p,1]$ is a continuous mapping from a closed and convex set into itself, it admits at least one fixed point, $\mathcal{B}(H^*)=H^*$, ensuring that *a symmetric equilibrium in pure strategies exists.*[8] The existence of a pure strategy equilibrium is crucial. The definition and existence of equilibrium in mixed strategies would be quite problematic in our setup because consumers treat probabilities in a non-linear fashion. We thus restrict analysis to pure strategies only.

That $\mathcal{B}(H)$ is increasing in H hinges on the positive externality: the more customers that 'wait', the higher the product availability in period 2. That $dq_2/dH>0$ is a bit counterintuitive at first, but quite clear on hindsight. Consider a small increase in H, i.e., a few consumers switch from 'buy now' to 'wait'. Some units that would have been purchased with probability q_1 are now purchased with probability $q_2\le q_1$. On a first-order approximation, the demand in period 2 increases by q_2, the supply increases by $q_1\ge q_2$, and the net effect is an increase in the availability.

We distinguish three regimes, depending on the abundance of supply. In the first regime all customers purchase with probability one. In the second regime, there is only rationing among those that decide to wait. In the third regime, all customers may experience rationing. The following result establishes this claim and characterizes equilibrium in each regime.

PROPOSITION 5. *Given (λ,F) and (Q,p,d), there are three regimes:*

I. ***Abundant supply****. If $Q\ge\lambda\bar{F}(p(1-d))$, then $q_1=q_2=1$, and the best response threshold is constant for all H and given by*

$$\mathcal{B}=\min\left\{p\cdot\frac{1-(1-d)e^{-s(r(d)t)}}{1-e^{-s(r(d)t)}},1\right\}. \tag{8}$$

There is a unique equilibrium given by $H^*=\mathcal{B}>p$.

II. ***Intermediate supply****. If $\lambda\bar{F}(p)<Q<\lambda\bar{F}(p(1-d))$, then $q_1=1$, $q_2\in(0,1)$, and the best response threshold is given by*[9]

$$\mathcal{B}(H)=\min\left\{p\cdot\frac{1-(1-d)e^{-s(\ln 1/q_2+r(d)t)}}{1-e^{-s(\ln 1/q_2+r(d)t)}},1\right\}. \tag{10}$$

There is at least one equilibrium solving $\mathcal{B}(H^*)=H^*>p$.

III. ***Limited supply****. If $Q\le\lambda\bar{F}(p)$, then $\mathcal{B}(H)=p$ on $H\in[p,F^{-1}\left(1-\frac{Q}{\lambda}\right)]$. We have that $H^*=p$ is always an equilibrium, but other equilibria with $H^*>p$ may exist.*

[8] We verify that $\mathcal{B}(H)\in[p,1]$. Moreover, λ_1, λ_2 are continuous functions of H, and q_1 and q_2 are continuous functions of λ_1 and λ_2, respectively.

[9] The equilibrium condition, $\mathcal{B}(H)=H$, can be rewritten as the fixed point problem:

$$q_2=1-\frac{\lambda\bar{F}(p(1-d))-Q}{\lambda\bar{F}(p(1-d))-\lambda\bar{F}\left(p\cdot\frac{1-(1-d)e^{-s(\ln 1/q_2+r(d)t)}}{1-e^{-s(\ln 1/q_2+r(d)t)}}\right)},\quad 0\le q_2\le 1. \tag{9}$$

Note that $\mathcal{B}(H) = p$ if and only if $q_2 = 0$. Thus, the equilibrium $H^* = p$, implies a congestion of 'buy now' customers facing $q_1^* = Q/\lambda\bar{F}(p) \leq 1$, and those that must wait face $q_2^* = 0$. Any equilibrium with $H^* > p$ exhibits $q_1^* = 1$ and $q_2^* > 0$. Still, if the price discount is small, $d \to 0$, or the future is far, $r(d)t \to \infty$, then $\mathcal{B} \to p$ and the equilibrium is eventually unique and close to p.

4.3 Uniqueness and Equilibrium Selection

A general condition for equilibrium uniqueness is $\mathcal{B}'(H) < 1$ at all points where $\mathcal{B}$ is differentiable. This condition is trivially met if the supply is abundant ($\mathcal{B}$ is constant). The condition is also met if the supply is intermediate but close to abundant. Indeed, as (17) shows, dq_2/dH, and hence $\mathcal{B}'(H)$, is proportional to $(\lambda_2 + \lambda_1 - Q)$. Hence, if $Q \to \lambda\bar{F}(p(1-d)) = \lambda_1 + \lambda_2$, then q_2 becomes insensitive to H, $\mathcal{B}(H)$ is quite flat, and the equilibrium unique. That $\mathcal{B}(H)$ is relatively flat reduces the strategic burden on our consumers.

If the supply is not close to abundant, however, q_2 might be quite sensitive to H and the equilibrium might not be unique. Lack of uniqueness is not due to behavioral effects. Osadchiy and Vulcano (2010) and more recently Correa et al. (2011) observe multiple equilibria under DEU and provide sufficient conditions for uniqueness. The following examples show multiplicity of equilibria under DEU and dPTT. Throughout, we set $\lambda = 1$, $F \sim Beta(0.6, 0.6)$, $p = 0.6$ and $d = 0.8$.

Example 1. DEU consumers [$\beta = 1$, $\mu = 0$, $\rho = 0.15$]. If supply is intermediate, $Q = 0.45$, then $\mathcal{B}(H) = H$ admits three solutions, namely, $H_1^* = 0.765$, [$q_1 = 1$ and $q_2 = 0.30$], $H_2^* = 0.961$, [$q_1 = 1$ and $q_2 = 0.50$]; and $H_3^* = 1$ [$q_1 = 1$ and $q_2 = 0.56$].

Example 2. DEU consumers with no discounting [$\beta = 1$, $\mu = 0$, $\rho = 0$]. If supply is limited, $Q = 0.4$, then $\mathcal{B}(H) = H$ admits three solutions, namely, $H_1^* = p = 0.6$, [$q_1 = 0.94$ and $q_2 = 0$], $H_2^* = 0.964$, [$q_1 = 1$ and $q_2 = 0.43$]; and $H_3^* = 1$ [$q_1 = 1$ and $q_2 = 0.50$].[10]

Example 3. dPTT consumers with no discounting [$\beta = 0.6$, $\mu = 0$, $\rho = 0$]. If supply is limited, $Q = 0.4$, then $\mathcal{B}(H) = H$ admits three solutions, namely $H_1^* = p = 0.6$ [$q_1 = 0.94$ and $q_2 = 0$], $H_2^* = 0.643$ [$q_1 = 1$ and $q_2 = 0.01$], and $H_3^* = 0.789$ [$q_1 = 1$ and $q_2 = 0.23$].

When the supply is not abundant and the game admits multiple equilibria, the game becomes one of coordination. Indeed, all equilibria can be Pareto ranked according to H^*, and the equilibrium with highest H^* is Pareto dominant. Recall that q_2 increases with H. Intuitively, the equilibrium with highest q_2^* improves the payoff of those who always wait, and leaves unaffected (under intermediate supply) or may improve (under limited supply) the payoff of those who always buy now.

[10] Under DEU and limited supply, $H^* = 1$ can be an equilibrium if $\frac{1-p}{1-p(1-d)} \leq e^{-\rho t} \frac{Q}{\lambda\bar{F}(p(1-d))}$. The more patient the consumers, the more likely this condition will be met. This same condition is not sufficient under dPTT, as Example 3 illustrates, but the intuition that patience fosters more waiting still holds.

Naturally, our selection criteria in the presence of multiple equilibria is to *choose the one with highest H^*.*

PROPOSITION 6. *Let H^* and H'^* be two equilibria. If $H^* > H'^*$, then H^* Pareto-dominates H'^*.*

We verify that the equilibria of the fluid model are good approximations of the equilibria of a stochastic demand model. To do so, we numerically analyzed a stochastic demand model (approximating the fluid model), in which the total number of customers is a Poisson random variable with rate λ. In that model, iterative calculations of the best response rapidly converge to a fixed point. The difference between the buy-now equilibria thresholds computed under the stochastic and fluid models diminishes as the demand and capacity are scaled up.

5. Markdown Management

Following the usual approach for analyzing Stackelberg games, we have assumed so far that the triplet (Q,p,d) is fixed, and studied equilibrium consumer behavior under dPTT. We have shown that a symmetric equilibrium exists, it is often unique, or otherwise there is unique Pareto-dominant equilibrium. Henceforth, for given parameters (β,ρ,μ), we consider the mapping $H^*(A):[0,Q]\times[0,1]\times[0,1]\to[p,1]$, which associates the unique Pareto-dominant equilibrium to each (Q,p,d).[11] In view of (5) and (6), the goal of the seller is to find the selling arrangement that maximizes

$$R(H^*) = p\cdot\min\{\lambda\bar{F}(H^*),Q\} + p(1-d)\cdot\min\{\lambda(F(H^*)-F(p(1-d))),\max(Q-\lambda\bar{F}(H^*),0)\}. \quad (11)$$

We will compare the equilibrium associated with (β,ρ,μ) (denoted by H^{dPTT}) with the equilibrium associated with $(1,\rho,0)$ (denoted by H^{DEU}). In particular, we will analyze the best response of the seller to each type of consumers.

Intuitively, the seller wants to be in the region of intermediate supply, or its frontiers, but not in the interior of regimes I and III. On the limited supply side, the seller will surely avoid a situation resulting in $q_1^* < 1$. The seller can consider multiple options, but one that will surely increase revenue is to increase prices, and offer no discounts, until the demand equals to supply.[12] On the abundant supply side, it might be optimal to set $Q=\lambda_1+\lambda_2$, but not to increase the quantity beyond this point; equivalently, the retailer may perish the excess inventory if it is priced so high

[11] The mapping is continuous whenever the equilibrium is unique. It ceases to be continuous if a new Pareto-dominant equilibrium appears.

[12] Formally, suppose that $H^*(Q,p,d)=p$ and $Q<\lambda\bar{F}(p)$ (i.e., $q_1^*<1$ is the unique equilibrium). Let $p'=F^{-1}(1-Q/\lambda)$. It is not difficult to see that the combination $A'=(Q,p',0)$ induces $H^*(A')=p'$ as the unique equilibrium. Indeed, A' results in $Q=\lambda\bar{F}(p')$ and $H^*(A')=p'$ (with no discount we have that $\mathcal{B}(H)$ is constant and the equilibrium unique). Because in A' the same quantity is sold at higher prices, we have that $R(H^*(A'))>R(H^*(A))$.

that it will remain unsold anyway[13]. Clearly, no new customers are served and the revenues remains the same. Thus the sellers best response is to be in the interior of the intermediate supply region, or its frontiers.

Given Q and p the seller's markdown optimization problem is to select d, that maximizes

$$R^{dPTT}(Q,p) = \max_d \big\{ p \min\{\lambda \bar{F}(H^{dPTT}), Q\} \\ + p(1-d)\min\{\lambda(F(H^{dPTT}) - F(p(1-d))), (Q - \lambda\bar{F}(H^{dPTT}))^+\}\big\}. \qquad (12)$$

To compare with DEU, we define

$$R^{DEU}(Q,p) = \max_d \big\{ p \min\{\lambda \bar{F}(H^{DEU}), Q\} \\ + p(1-d)\min\{\lambda(F(H^{DEU}) - F(p(1-d))), (Q - \lambda\bar{F}(H^{DEU}))^+\}\big\}. \qquad (13)$$

In both cases, of course, H is the equilibrium mapping associated with (Q,p,d).

We claim that the dPTT model yields larger markdowns at the optimum compared with the DEU model. To substantiate this claim, the following proposition analytically proves this result on a restricted part of the parameter space[14], and the numerical illustrations that follow support it for a broader range of parameter values.

Proposition 7. *Assume $u \sim U[0,1]$, $\rho t < 1$, and $\mu = 0$. If $Q > \lambda\left(1 - \frac{1}{2}p(1+e^{-\rho t})\right)$, then $d^{DEU} = \frac{1}{2}(1-e^{-\rho t})$ and $\frac{\partial R^{DEU}}{\partial d}|_{d=d^{DEU}} = 0$. However,*

$$\frac{\partial R^{dPTT}}{\partial d}|_{d=d^{DEU}} > 0.$$

We complement this theoretical result with the numerical verification that $\frac{\partial R^{dPTT}}{\partial d}|_{d=d^{DEU}} > 0$ for $(\beta,\mu) \in [0,1) \times [0,6.5]$. We consider an illustrative base case with $\lambda = 1$, $u \sim U[0,1]$, $Q = 0.625$, and $p = 0.5$. The quantity Q ensures that the seller is in the abundant or intermediate supply regime. We set the wait time until a markdown to $t = 3$, which corresponds to 9 weeks (meaning that a unit of time is 3 weeks, which is an important element of our elicitation experiment, see §6). This is consistent with Bils and Klenow (2004) who found the median price duration of 4.3 months (18 weeks) for a broad panel of consumer goods and services. We set the time discounting rate $\rho = 0.13$, and the reference discount parameter $d_0 = 0.5$ (see §6 for details). For the dPTT case we first use parameters $\beta = 0.9$ and $\mu = 1.95$ (observations i)-iv) below) and the extend the parameters study region to $(\beta,\mu) \in [0,1] \times [0,6.5]$ (observation v). Recall that the DEU model corresponds to $\beta = 1$ and $\mu = 0$.

[13] Formally, one can redefine our selling mechanism as if the retailer is endowed with $\bar{Q}$ units of inventory, of which it decides to perish $\bar{Q} - Q$ units and no extra cost or salvage value.

[14] The assumption $\mu = 0$ is required for tractability of the derivative bound. The assumption $\rho t < 1$ is consistent with the experimental data (see Baucells et al. 2009, and below).

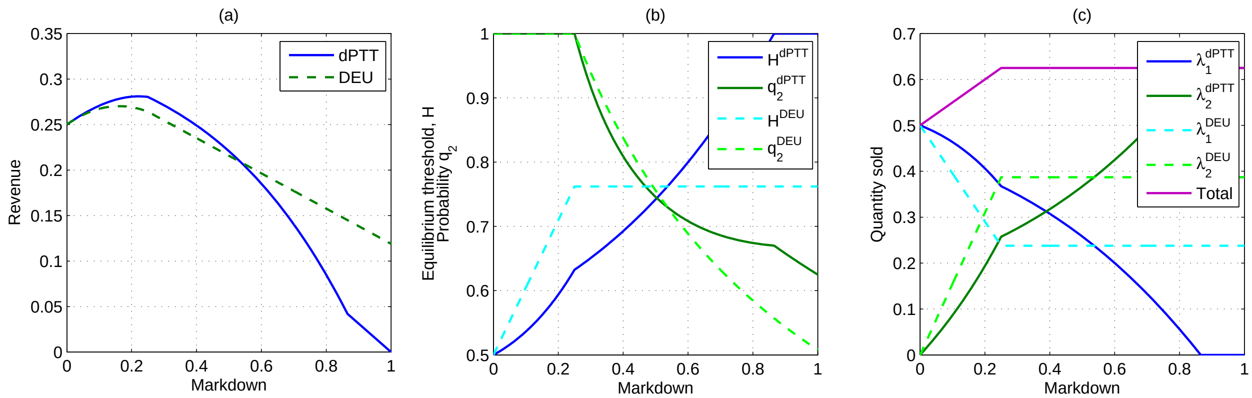

Figure 1 **(a) Revenue, (b) Equilibrium threshold H and probability q_2, (c) Buy-now λ_1, buy-later λ_2, and total sold quantity as a function of markdown d, under dPTT $(\beta = 0.9,\ \mu = 1.95)$ and DEU models. Parameters:** $Q = 0.625,\ p = 0.5,\ t = 3,\ \lambda = 1,\ u \sim U[0,1],\ d_0 = 0.5,$ **and** $\rho = 0.13$.

i) **Optimal revenue under dPTT is greater than under DEU.** Behavioral anomalies featured in the dPTT model allow retailer to extract more revenue. In our example, the optimal revenue under the dPTT model exceeds the optimal revenue under the DEU model by 4.1% (0.281 vs 0.270, Figure 1, a). Note that DEU benefits from price discrimination (the DEU revenue beats the fixed price policy by 8%). Consumer impatience as characterized by ρ is crucial for retailer's ability to price discriminate. This is consistent with von der Fehr and Kuhn (1995) who found that retailer must be more patient than consumers for effective price discrimination. Due to subendurance consumers become more patient as d increases reducing effectiveness of price discrimination (see thresholds H^{dPTT} and H^{DEU}, Figure 1, b). If $d > 0.85$ all consumers choose to wait under dPTT and revenues decrease.

ii) **Optimal markdown under dPTT is greater than under DEU.** The higher revenue under dPTT is due to more aggressive markdowns offered by dPTT than by DEU. While the DEU achieves optimal revenue at 16.1% markdown, the optimal markdown under dPTT is 22.2%. In both models optimal markdowns ensure that the probability of receiving an item in the second period $q_2 = 1$, i.e., every consumer with benefit of consumption $u \geq p(1-d)$ receives an item (Figure 1, b). For both dPTT and DEU the optimal discount increases with consumer impatience.

iii) **At the optimum dPTT sells more units at tag price than DEU.** In our example the buy-now threshold $H^{DEU} \geq H^{dPTT}$ for levels of $d \leq 0.53$ (Figure 1, b). In that case, the number of items sold at tag price (buy-now) is greater under the dPTT model. At the optimum, the dPTT model sells 0.391 units (64%) at tag price and 0.220 units (36%) at discount. The respective quantities for the DEU model are 0.332 units (57%) and 0.248 units (43.0%).

iv) **dPTT sells more units in total than DEU.** The total number of units sold by dPTT

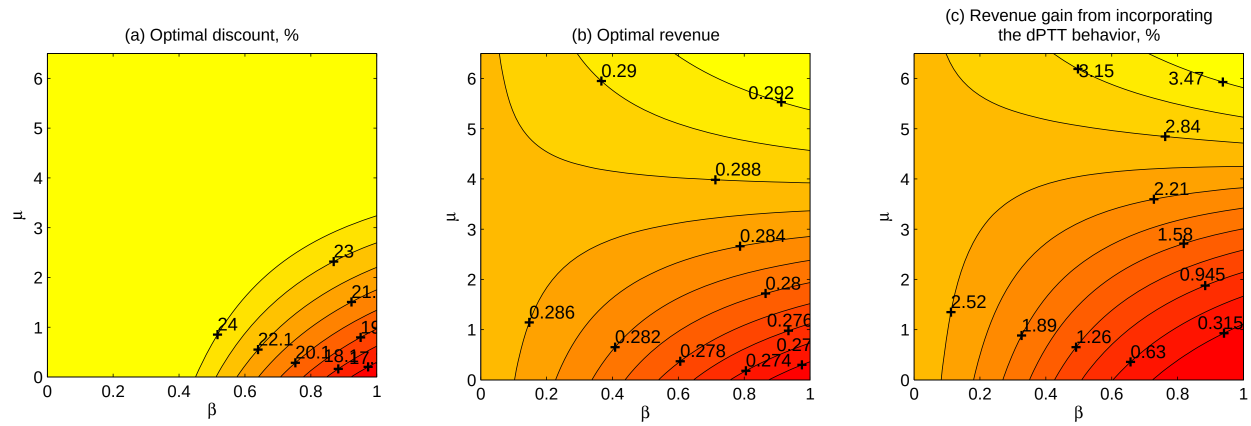

Figure 2 **(a) Optimal discount, (b) Optimal revenue, (c) Revenue gain from incorporating the dPTT behavior as a function of β and μ. Parameters:** $Q=0.625,\ p=0.5,\ t=3,\ \lambda=1,\ u\sim U[0,1],\ d_0=0.5,$ **and** $\rho=0.13.$

is equal to the total number sold by DEU for the same discount level. However, given that dPTT offers higher discount at the optimum, it also sells a larger number of products in total. In our example dPTT sells 0.611 units while DEU sells 0.535 units (gain of 11.2%, Figure 1, c). Note that under dPTT the seller could achieve the same revenue by stocking 0.611 units.

v) **dPTT delivers higher revenue as consumers deviate more from rationality.** We investigate the effect of β and μ on the optimal discount and revenue. Recall, that the DEU model corresponds to $\beta=1$ and $\mu=0$, thus, $\beta<1$ and $\mu>0$ are considered to be deviations from rationality. We find that a larger deviation from rationality results in a greater discount and higher revenue at the optimum. For example, if $\beta=0.5$ and $\mu=3$, the optimum discount is 25% (Figure 2, a) and the optimum revenue is 0.286 (Figure 2, b).[15] If μ is small (corresponding to psychological distance $\sigma\le 1$), deviations from rationality in β and μ complement each other, i.e., a deviation in either parameter results in a higher revenue. However, if μ is large (corresponding to $\sigma>1$), a smaller deviation in β increases attractiveness of the buy-now option even further which in turn drives revenue higher.

Correctly accounting for the dPTT behavior and incorporating it into pricing results in substantial revenue increase over using the traditional pricing models based on the DEU model. The revenue gain can be as high as 3% in some cases (Figure 2, c).

The explanation for superior performance of the dPTT model lies in the decreasing sensitivity to psychological distance (concave increasing $s(\sigma)$) and the subendurance effect ($r(d)$, see §3). Around zero, the sensitivity to psychological distance is high implying that customers who buy-now tend to

[15] Compared with the $\beta=0.9$, $\mu=1.95$ case, $\mu=3$ increases attractiveness of they buy-now option. The optimum discount platoes at the inventory clearing level. Further increasing μ, the increase in revenue is due to the higher fraction of buy-nows.

keep their choice if markdown is increased, this effect becomes stronger if β is low. Subendurance implies that $r(d)$ is decreasing in d making customers more patient for large markdowns and less patient otherwise, this effect is amplified by μ. Collectively this allows a retailer to offer higher markdowns without suffering a loss in full price sales if consumers' deviation from rationality is substantial, i.e., if β is low and μ is high.

The practical importance of the above results, however, clearly depends on whether in reality β is indeed sufficiently low and μ is sufficiently high. To shed light into that in the remainder of the paper we describe a behavioral experiment that we used to estimate our model parameters and consequently establish its practical significance.

6. Experimental Estimation of the dPTT Paremeters

The goal in this section is to estimate the dPTT model parameters. For baseline time discount parameter we used $\rho = 13\%$ which is the average discount rate of 18% for monetary gains between €50 and €100 from (Baucells et al. 2009) adjusted for the fact that a unit of time in their study was a month and in ours it is 3 weeks ($18 \times 3/4 \approx 13$). Interestingly, fitting the DEU model to our wait-or-buy data also results in $\rho = 13\%$. For the reference discount parameter we used $d_0 = 50\%$ which we obtained through an online survey with $N = 32$ student participants from Canada. We asked them: "Think about an end-of-season sale (markdown) at a retail store – such as Boxing day, for example. What is the percentage price discount that first comes to mind?" The average response was 51%, both the mode and median were 50%. For the sensitivity to discount μ, and the sensitivity to psychological distance β we conducted an experiment described below.

6.1 Design of the Experiment

To estimate these parameters, it suffices to collect consumer wait or buy choice data for various discount levels d, and probabilities of product availability q. The dPTT model allows us to hold the benefit of consumption u, the tag price p, and the time delay t constant.

The choice data can be collected in two ways. An intuitive approach is to collect the *binary choice* data, i.e., present subjects with combinations of (d, q) and ask if they would buy or wait. The challenge of this approach is that to have a good coverage of input parameter space each subject would have to answer dozens of nearly identical questions, causing subjects fatigue and dis-engagement, which are known to significantly lower the quality of elicitation. An alternative approach is to use *choice-lists*, e.g., present subjects with blocks of questions each containing a list of binary wait-or-buy questions for different values of q. d is held constant in each block, and varied across blocks. Such an approach has been shown to increase the quality of elicitation in situations where there is an implicit indifference point (this will be shown for our data soon). See the well-known paper by Holt and Laury (2002) for an example of using choice-lists, and see Bodily and Pfeifer (2010) for an experimental illustration for how using choice-lists improves the quality of elicitation when there exists an implicit indifference point.

In the experiment we set the benefit of consumption $u = 250$, and the tag price $p = 200$. The time delay is set at three weeks, which we normalized as $t = 1$.

The buy later option was varied over a wide range of probabilities of product availability $q = 10\%, 20\%, \ldots 90\%$ and discounts $d = 5\%, 15\%, 25\%, 50\%, 75\%$. The selected discounts are based on the evidence from Elmaghraby et al. (2014, Table 1) who documented that inventory liquidators in practice recover 27 ± 5 cents on a dollar of tag price, hence discounts larger than 75% seem ineffective. Thus, each subject saw five choice-list questions, one for each d, with nine q values in each list. The five lists appeared in random order, see Figure 7 (a) in the appendix for the screenshot.

As a reliability test, and after answering the choice lists, subjects answered five binary choice questions, one from each choice list, drawn and ordered randomly as well, see Figure 7 (b).

To make the elicitation incentive-compatible we determined the payments using a standard random incentive mechanism (RIS). Specifically, we implemented the Prior Incentive Scheme (Prince) (Johnson et al. 2014). The Prince method refines the random incentive system, arguably first suggested in Savage (1954), to improve incentive compatibility and reduce various problems in preference elicitation that has been documented by previous researchers. RIS compensates subjects according to the results from a scenario randomly chosen after an experiment has concluded. Under the Prince method, the scenario for potential compensation is randomly assigned to a subject *before* the experiment, but is unknown until conclusion of the experiment. The scenario is provided to the subjects in a tangible/physical form (usually a sealed envelope), and subjects' answers are framed as instructions to the experimented about how to implement the real choice situation contained in the envelope.

6.2 Subjects and Procedure

Subjects, 64[16] business school students, were recruited through online system to participate in a decision-making experiment that promised earnings of a minimum of \$5 and a maximum of \$200. Upon arrival subject picked physical sealed envelopes and experimental instructions (see appendix). The instructions contained the following description of the decision situation:

[16] We have 45 (d, q) combinations, and we ensured that all 45 combinations were assigned to some subject's envelope.

`Suppose that you went to a retail store and saw a product that you know you can resell for $250 at any time. The product was priced at $200 (two hundred dollars), so you picked the product from the shelf and were about to purchase. However, then you started thinking that in three weeks from today this product may be marked down. Thus the question was: should you buy the product now or wait for the markdown?`

From reading the instructions subjects learned that the sealed envelopes contained two numbers: markdown percentage and probability of product availability. They further learned that two of them will be selected at the end of the experiment. The envelope of a selected subject will be privately opened and the scenario in the envelopes will be played as per the choices she will make during the experiment. That is, if in that scenario she chose to "buy now" then (s)he will receive the "buy now" payment, $50, immediately. If (s)he chose to "wait for the markdown" then (s)he would come again to see the experimenter in three weeks to learn about the product availability (determined by whether a U[1,100] random integer is less or equal to q) which will in turn determine her payoff: zero if the product is unavailable, or $250 - 200 \times (1-d) \in [\$60, \$200]$.

As per the instructions, two subjects were randomly selected, ID49 (male) with $d = 25\%$ and $q = 50\%$, and ID11 (female) with $d = 25\%$ and $q = 80\%$. ID49's response to the $q = 50\%$ question in the $d = 25\%$ choice-list was to "buy now" thus he was awarded $50 and left. ID11's response to the $q = 80\%$ question in the $d = 25\%$ choice-list was to "wait for the markdown" thus she left the experiment with no payment, but in three weeks came again to see the experimenter. The drawn random number was $15 \leq 80 \equiv$ (her q), which meant that the product was available. Thus she "bought it" for $200*(1-.25)=$150 and immediately resold to the experimenter for the surplus of $100. That concluded our experiment.

6.3 Structure of the Data and Initial Checks

The experimental data consists of the series of wait-or-buy decisions that subjects made for different (d, q) combinations. However, within each choice-list such data are not independent, particularly if the choice-lists reveal the indifference points[17]. An indifference point would imply that within each choice-list as q's increase, subjects switch from selecting "buy" for low q's to selecting "wait" for high q's, and they do so only once (i.e., do not switch back to "buy" at even higher q's). Then somewhere between the highest "buy" q and the lowest "wait" q is the probability at which, for that specific d, the subject is indifferent between buying and waiting.

[17] The ability to reveal an indifference point is an advantage of choice lists over the binary choice. The choice lists are designed to cover enough possibilities to reveal an indifference point

Our data shows overwhelming support for the existence of indifference points: 62/64 subjects exhibited such an indifference-point pattern in *all* choice-lists, one subject switched more than once in a single choice-list, and one subject, ID56, exhibited the behaviour that is largely inconsistent with the notion of indifference point. We believe that (s)he mis-understood the task; this is further supported by the consistency analysis that follows. Thus we removed subject all subject ID56 data, and for for the rest defined indifference points as the mid-value between the highest "buy" q and the lowest "wait" q. Whenever a choice-list had all "buys" the indifference points was defined as 95%, and conversely, in a choice-list with all "waits" the indifference point was defined at 5% – in both cases these are the mid-points between the corresponding subject's answer and the boundary of the $[0,1]$ interval for probabilities. This led to 315 indifference points. We note that mid-point indifference points were also used in Holt and Laury (2002) and Bodily and Pfeifer (2010).

The binary choice data were used to verify within-subject consistency by comparing the wait-or-buy decision made in a binary-choice question to the corresponding decision made in the respective choice-list. This analysis also reveals formidable consistency: 67% of our subjects were consistent in *all* decisions, 23% of subjects were consistent in all but one choice, and only one subject, the familiar ID56, was inconsistent in more than three. That supported our decision to exclude his/her data from the analysis. The remainder is a set of highly consistent wait-or-buy data on 63 usable subjects, which we next use to estimate the parameters of our consumer utility model.

6.4 Parameter Estimation

Given the structure of our data we designed the estimation procedure so as to minimize the deviations between the observed and implied indifference points. Note that we purposefully did not use a somewhat more intuitive maximum likelihood estimation for the binary choice data, because the existence of the indifference points for most subjects implies that many of the binary choice data points are not independent. Indeed, if for $d = 25\%$ one's indifference point is $q = 65\%$, then the only two independent binary choice data points are a "buy" for $d = 25\%, q = 60\%$ and a "wait" for $d = 25\%, q = 70\%$: all data points with $q < 60\%$ will have a "buy" decision, and those with $q > 70\%$ will have a "wait".

The choice-list data consists of a set of indifference pairs in the form of $(\tilde{d}_{ij}, \tilde{q}_{ij})$, where $\tilde{d}$ is the observed discount value and $\tilde{q}$ is the imputed indifference probability value as explained above, j is the index for the subject, and i is the index for the observation. In the context of our model, indifference between buying and waiting given a (d, q) pair and model parameters β, μ, ρ implies

$$u - p = (u - p(1-d)) \exp\left[-\left(\ln\frac{1}{q} + \rho e^{\mu(d_0 - d)} t\right)^{\beta}\right]. \tag{14}$$

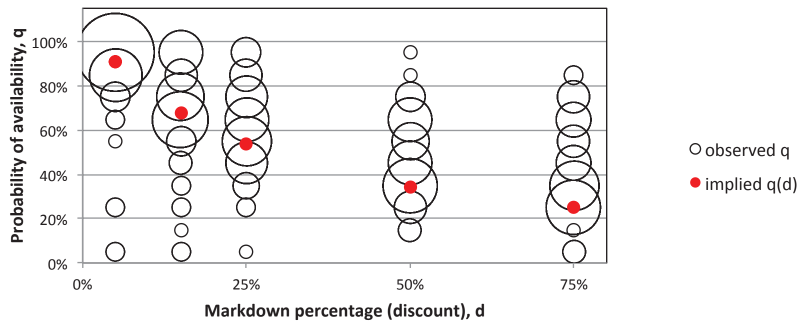

Figure 3 **Observed, $\tilde{q}_{ij}$ and implied $q(\tilde{d}_{ij})$ indifference probabilities.**

Since the expression on the left is independent of q and the expression on the right is a monotone function of q, for any d there exists the implied $q(d)$ for which equation (14) holds. Solving for $q(d)$ and rearranging we obtain that:

$$q(d) \equiv q(d)|_{\beta,\mu} = \exp\left\{-\left(-\ln\left(\frac{u-p}{u-p(1-d)}\right)\right)^{\frac{1}{\beta}} + \rho e^{\mu(d_0-d)}t\right\}. \tag{15}$$

Figure 3 presents the observed indifference probabilities for different discounts; the size of the bubble is proportional to the number of subjects with the same $(\tilde{d}_{ij}, \tilde{q}_{ij})$ combination. The Figure also presents the implied $q(d)$ values for the optimal fit. Finally, it highlights an important observation that motivates how we fit our model to this data: the data points (and hence the resultant errors in the estimation) are censored. Indeed, since the indifference probability cannot be larger than 100% or smaller than zero, for small discounts the estimated errors will be censored on the left, and for large discounts, on the right. As argued by Powell (1984), in a situation with censored observations and errors, the least-absolute-deviation (LAD) criterion is more appropriate than the standard least-squares (LS) estimation. Thus we measure the goodness-of-fit between the implied and observed probabilities for a given discount and model parameters by the absolute difference:

$$LAD_{ij}(\cdot)|_{\beta,\mu} = |(\tilde{q}_{ij} - q(\cdot))\,|_{\beta,\mu}|\,. \tag{16}$$

We select the parameters of the pooled model such that to minimize the total LAD over all subjects and lists, i.e., by solving the following optimization model:

$$\min_{\beta,\mu}\left[\sum_{i,j} LAD_{ij}(\tilde{d}_{ij})|_{\beta,\mu}\right].$$

Individual models can be similarly defined for each subject j by taking a sum over i only. Note that similarly to how least squares regression is interpreted as a conditional mean, the LAD regression is interpreted as a conditional median (Powell 1984). As per the prior literature, we constrained $\mu \geq 0$ and $\beta \in [0,1]$. The estimation was performed in VBA using the multi-start generalized-reduced-gradient (GRG) engine in the Premium Solver Platform.

6.5 Estimation Results

Figure 4 presents the estimated β and μ parameters. Each of the 63 $\times$'s on Figure 4 represents the estimated individual β_j, μ_j pair using the LAD method based with the choice-list data[18]. The circle is the mean of those individual estimates, $\beta = 0.73, \mu = 2.17$, and the square is the median of individual estimates: $\beta = 0.91, \mu = 1.50$.

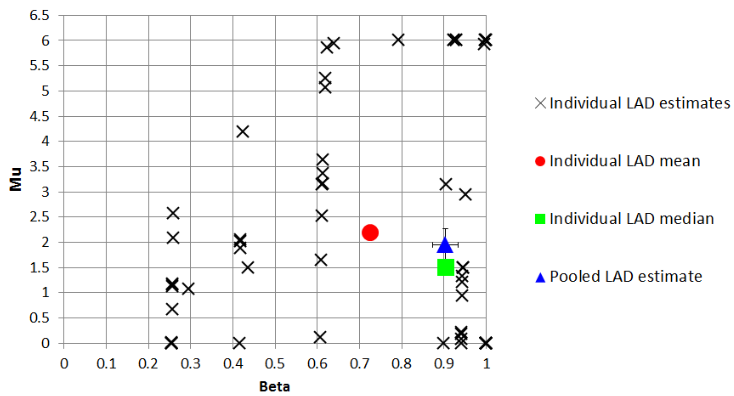

Figure 4 Estimates of consumer utility parameters β and μ.

The triangle on Figure 4 represents the pooled LAD estimate: $\beta = 0.9, \mu = 1.95$. The error bars correspond to two standard deviations (errors) for the corresponding estimated parameter. Given that our model is highly-nonlinear, and hence its error structure is unclear, we calculated standard errors by bootstrapping (the jackknife approach was used, Efron (1979), Boos and Stefanski (2013), Chapter 10). The estimated errors are 0.015 for β and 0.162 for μ. It is thus evident that the subjects significantly deviated from the fully rational estimates ($\beta = 1, \mu = 0$).

Two additional observations are evident from the Figure. First, the pooled estimate and the median of individual estimates are remarkably consistent. The mean is slightly off, which is not surprising given that the parameters are censored at $\beta = 1$. Second, the total degree of irrationality, the distance from the $\beta_j = 1, \mu_j = 0$ corner, seems to be somewhat similar for many subjects, but for some it reveals in being irrational with respect to the non-linearity in psychological distance, while for others with respect to sensitivity to the magnitude of the discount (subendurance).

To conclude the experimental part of our paper, we designed and executed an experiment to measure parameters of the consumer value function in our model. The estimated sensitivity to psychological distance parameter $\beta \approx 0.9$, and the sensitivity to discount magnitude $\mu \approx 1.95$.

[18] For 7/63 subjects the individual estimate is $\beta_j = 1, \mu_j = 0$ (the lower left corner of the Figure), i.e., that consistent with the fully rational expected utility behavior. Generally, 23/63 subjects have estimated $\beta_j = 1$, i.e., the constraint on β was binding. It is worth noting that allowing $\beta > 1$, the average $\beta = 0.99$ and for 53/63 subjects $\beta_j < 1.3$.

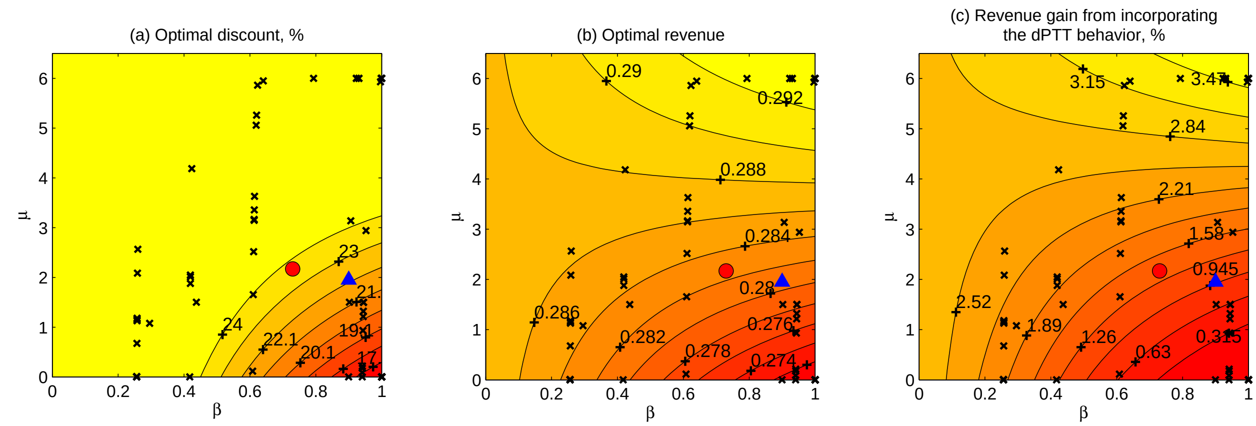

Figure 5 **Experimental results and optimal pricing: (a) Optimal discount, (b) Optimal revenue, (c) Revenue gain from incorporating the dPTT behavior as a function of β and μ.**
$\times$ **- individual estimates, ▲ - pooled estimate.**
Parameters of the pricing model: $Q = 0.625,\ p_h = 0.5,\ t = 3,\ \lambda = 1,\ u \sim$ **Unif**$[0,1]$**,** $d_0 = 0.5$ **and** $\rho = 0.13$**.**

Despite the substantial heterogeneity in the individual estimates, the remarkable consistency of the median and pooled estimates suggests that the estimated parameters are quite robust.

6.6 Optimal Markdowns with the Estimated Parameters

Figure 5 summarizes the combination of our numerical exploration and experimental calibration. Recall that the optimal DEU discount is 16%. If $\beta = 0.9$, $\mu = 1.95$ (pooled LAD estimate), the optimal dPTT discount is 22%. Compared to the DEU optimum, implementing dPTT pricing increases revenue by 1.0% in this case. If $\beta = 0.73$, $\mu = 2.17$ (individual LAD mean) the optimal discount is 24% and the revenue lift is 1.4%. Observe further that the optimal discount set at the inventory clearing level (25%) is applicable to a wide range of (β, μ) combinations. In fact, it is optimal for 56% of the subjects (35 out of 63). In that sense, the optimality of dPTT pricing is robust with respect to parameters describing anomalies in consumer behavior.

Figure 6 gives insight to the larger discounts and highlights the relative contributions of the decreasing sensitivity to psychological distance and subendurance to the optimal markdown. It plots the marginal benefit of increasing markdown d (i.e., selling an extra item subject to inventory constraint) vs. the marginal cost associated with larger d (i.e., lower markdown price and switching from tag price to markdown purchases).

The optimal markdown balances the marginal benefit with the marginal cost. The marginal benefit is linear decreasing in d; it is the same for both models and drops to zero at $d = 0.25$ due to the inventory constraint. The marginal cost is increasing in d for both models and in fact is linear for DEU. Observe that in all cases on Figure 6 the revenue under DEU (dashed line at the top) is maximized when the two bold lines intersect: the marginal cost = marginal benefit logic.

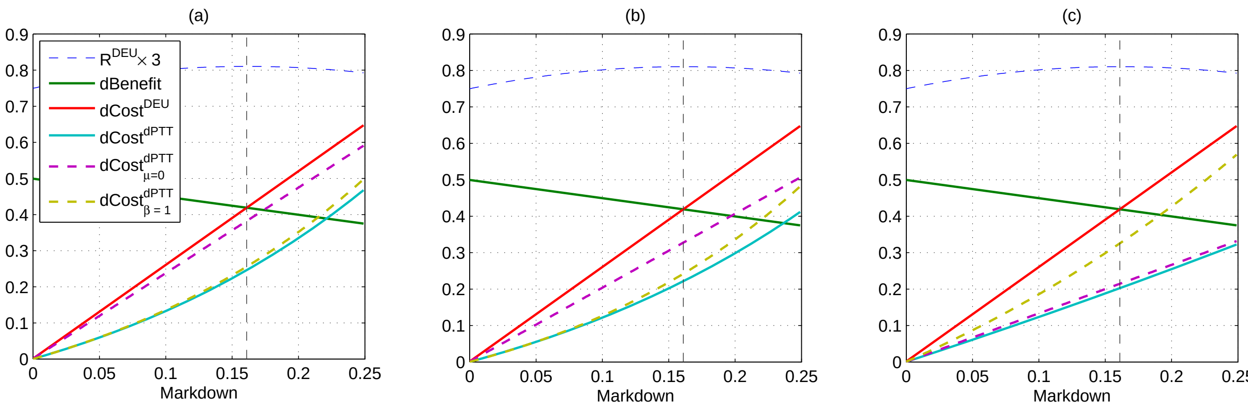

Figure 6 **dPTT drivers of optimal markdown for (a)** $\beta = 0.9,\ \mu = 1.95$; **(b)** $\beta = 0.73,\ \mu = 2.17$; **(c)** $\beta = 0.25,\ \mu = 1$. **Parameters of the pricing model:** $Q = 0.625,\ p_h = 0.5,\ t = 3,\ \lambda = 1,\ u \sim$ **Unif**$[0,1], d_0 = 0.5$ **and** $\rho = 0.13$.

Under the dPTT the marginal cost is non-linear. An immediate observation is that the marginal cost is smaller under dPTT hence it intersects the marginal benefit line at a higher discount. Setting $\beta = 1$ isolates the subendurance effect, and $\mu = 0$ isolates the effect of decreasing sensitivity to psychological distance. At the estimated median or mean parameters the larger markdown result is driven mostly by subendurance, see Figures (a) and (b). But at $(\beta, \mu) = (0.25, 1)$ (there are 5 subjects in that neighborhood, Figure 4), the effect is driven mostly by decreasing sensitivity to the psychological distance, see Figure (c).

7. Conclusions

The importance of markdown management to modern retailers is hardly of a question: nearly 1/3 of unit sales and 1/5 of dollar sales are generated at markdowns (Smith and Achabal 1998, Agrawal and Smith 2009). Further, with retailers' net profit margins being approximately 3%, each percent of extra markdown revenue translates into major profit increases. The proliferation of markdowns, however, fueled strategic waiting: effectively every time a consumer enters the store (s)he mentally "solves" a wait-or-buy problem – should (s)he buy the item now or wait for a possible markdown. Being aware of the behavioral regularities surrounding this decision, and incorporating them into markdown management offers substantial revenue opportunity for retailers.

In is paper we study this fundamental wait-or-buy problem from a unique, behavioral, perspective. The core idea of our study is that the wait-or-buy decision reflects a multi-dimensional trade-off between the delay in getting an item, the likelihood of getting it, and the magnitude of the price discount. Multiple studies in the decision analysis, psychology and behavioral economics, showed that all these trade-offs are prone to behavioral irregularities by which human decision makers deviate from the discounted expected utility model used in the current literature.

We present behavioral preference foundations (axioms) that support a modification of the discounted utility model. Our axioms capture three behavioral anomalies widely documented in laboratory experiments, namely, the common ratio effect in risk perception, the common difference effect in time perception (a.k.a. hyperbolic discounting), and the magnitude effect in time discounting (a.k.a. subendurance). Key in our formulation is the concept of psychological distance. The result is a parsimonious modification of the discounted expected utility. We solve the consumer's wait-or-buy problem and embed it into the firm's markdown optimization problem. We calibrate the model parameters using experimental data and show that accounting for the behavioral anomalies results in substantially larger markdowns that the current literature suggests *and* leads to noticeable revenue gains.

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Appendix

Experimental Instructions

You are about to participate in an experiment in the economics of decision-making. There are no right or wrong answers; just express your preferences. By doing so you can earn a substantial amount of money that will be paid to you as is explained below. If you have a question at any time, please raise your hand and the experimenter will answer it; do not talk with one another for the duration of the experiment.

Overview of the experiment

The context of the experiment is the following:

`Wait or Buy? Suppose that you went to a retail store and saw a product that you know you can resell for $250 at any time. The product was priced at $200 (two hundred dollars), so you picked the product from the shelf and were about to purchase. However, then you started thinking that in three weeks from today this product may be marked down. Thus the question was: should you buy the product now or wait for the markdown?`

As you entered the room you selected a sealed envelope. Do not open the envelope until the end of the experiment. The envelope contains two numbers: (1) the markdown percentage and the (2) likelihood that the product will be available when you visit the store again in three weeks. Either number could vary between 5% and 90%. As of yet, you do not know which two numbers are inside your envelop.

With the help of the experimental interface (see the screenshot and the link on the next page) you will give the experimenter the instructions whether you would like to buy the product now or wait for three weeks until the markdown for each possible combination of the markdown percentage and the likelihood of availability that could be inside your envelope. You will first be asked 5 questions, each containing a single markdown percentage and 9 different likelihoods of availability. Your answers to those questions will determine your pay. We will then ask you 6 additional questions each containing a single markdown percentage and a single likelihood of availability just to double-check your answers.

How you will be paid

At the minimum you will be paid $5 just for participation in this experiment. At the maximum you can earn over $200. We will determine how much you will earn as follows. At the end of the experiment today we will randomly select two participants. These selected participants will stay with the experimenter to complete the process; everyone else will collect their $5 and leave.

The experimenter (we) and each selected participant (you) will in private complete the following procedure. First, we will then open your envelope and find out your markdown percentage and the likelihood of availability. Second, we will look-up the Wait-or-Buy choice you made during the experiment for that specific markdown percentage and likelihood of availability.

• If your choice was Buy now then you will buy the product for $200 and immediately resell it to the experimenter for $250, keeping the remaining $250 - 200 = $50.

• If your choice was Wait for the markdown then in three weeks (i.e., on April 23, 2014) you will have to stop by Prof. [Removed by the authors for confidentiality] office to learn about the product availability.

—Product availability will be determined by drawing a random number between 1 and 100 (all numbers being equally likely) and comparing it to the likelihood of availability from your envelope. If the random number is less or equal to the likelihood from the envelope, the product will be determined to be "available" and otherwise "unavailable".

—If the product is available, then its price will be adjusted according to the markdown percentage from your envelope, you will pay the adjusted price, immediately resell the product to the experimenter for $250 and will keep the rest of the monies. For example, if the markdown percentage is 50% then the adjusted price will be $200 X 50% = $100, and after re-selling the product you will keep the remaining $250 100 = $150.

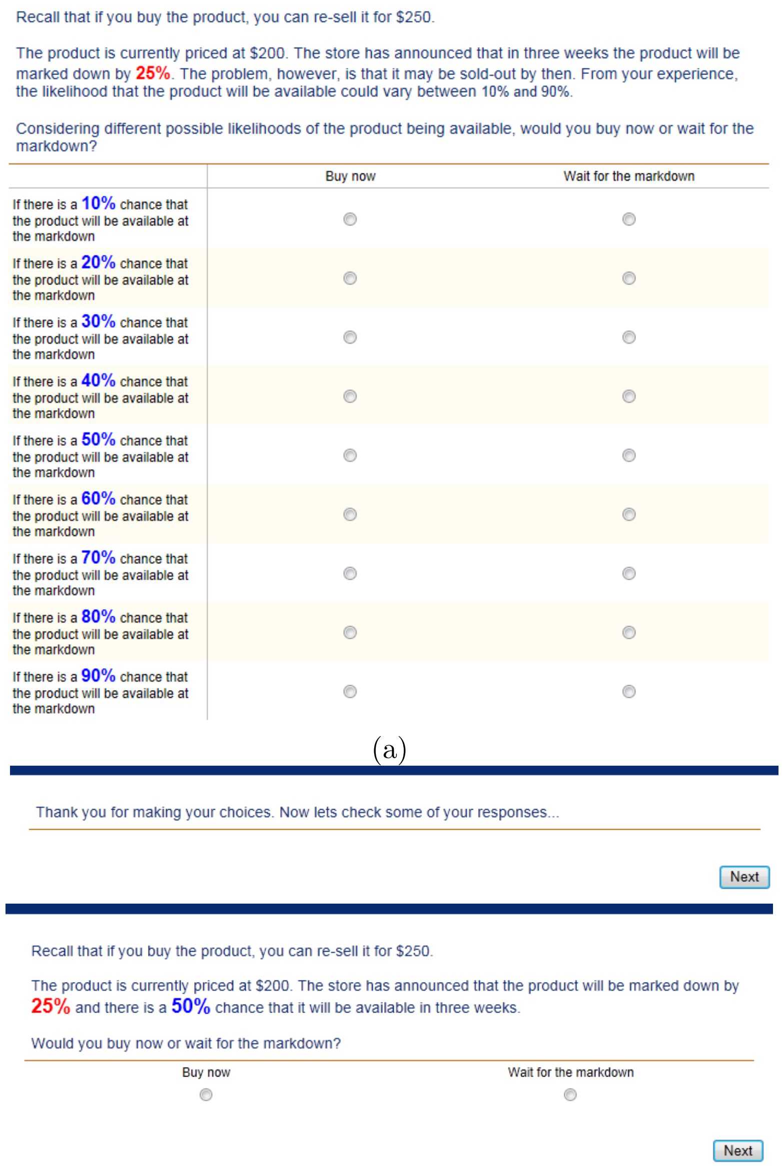
—If the product is unavailable, then you will have nothing to resell and thus there will be no additional money.

All moneys will be paid to you in cash. All decisions and earnings are confidential.

Screenshot of the experimental interface

[Removed by the authors to save space; it is identical to Figure 7 (a) except that the actual markdown percentage was replaced with XX to ensure that all values receive equal amount of subjects attention.]

Link: [Removed by the authors for confidentiality.]



(b)

Figure 7 **Screenshots for the choice-list (a) and binary choice (b) questions.**

Proofs

Proof of Proposition 1. It is routine to verify that V satisfies A1-A8. For the converse, we have that A1 implies that, for some arbitrary $a \in \mathbb{R}$, preferences are represented by a continuous function, $V_\tau(a-p,d,q,t)$. A2 implies that $V_\tau(x) = V_0(t-\tau, x_{-t})$. Hence, suffices to determine V_0. In A3.u, $u > 0$ does not depend on x and we set $a = u$.

By A5-A6 (Baucells and Heukamp 2012, Thm. 1) and A4, respectively,

$$V_0(u-p,d,q,t) = V_0(u-p,d,qe^{-r(d)t},0) = V_0(u-p(1-d),0,qe^{-r(d)t},0).$$

We apply A3.0 to conclude that V_0 takes constant value if either $u-p(1-d)=0$ or $qe^{-r(d)t}=0$, and normalize $V_\tau(\mathbf{0}_\tau)=0$, $\tau \geq 0$. By A3.p and A3.q, V_0 is strictly increasing in the first and third components. By A8 (Baucells and Heukamp 2012, Thm. 2), applies separately to gains and losses,

$$V_0(u-p(1-d),0,qe^{-r(d)t},0)=\begin{cases} v(u-p(1-d))\ w^+(qe^{-r(d)t}), & u-p(1-d)\geq 0 \\ v(u-p(1-d))\ w^-(qe^{-r(d)t}), & u-p(1-d)<0. \end{cases}$$

Combining the equalities and letting $s^+(\sigma)=-\ln w^+(e^{-\sigma})$ and $s^-(\sigma)=-\ln w^-(e^{-\sigma})$ yields (2). Remains to show that s^+ and s^- are concave.

For any choice of $\sigma \geq 0$, $\sigma', \sigma''>0$, let $t=0$, $q=e^{-\sigma}$, $\theta=e^{-\sigma'}$, and $q'=qe^{-\sigma''}$. Given are u, p, and d. If $u-p(1-d)>0$, then find $p' \in (p, u/(1-d))$ such that $(p',d,q,0)\sim(p,d,q\theta,0)$, or $s(\sigma+\sigma')-s(\sigma)=\ln\frac{v(u-p(1-d))}{v(u-p'(1-d))}$. If A7 holds, then $(p',d,q',0)\preceq(p,d,q'\theta,0)$, or $s(\sigma+\sigma'+\sigma'')-s(\sigma+\sigma'')\leq \ln\frac{v(u-p(1-d))}{v(u-p'(1-d))}$. Combining the two results yields $s^+(\sigma+\sigma'+\sigma'')-s^+(\sigma+\sigma'')\leq s^+(\sigma+\sigma')-s(\sigma)$.

If $u-p(1-d)<0$, then find $p'\in(0,p)$ such that $(p',d,q,0)\sim(p,d,q\theta,0)$, or $s(\sigma+\sigma')-s(\sigma)=\ln\frac{v(u-p(1-d))}{v(u-p'(1-d))}$. If A7 holds, then $(p',d,q',0)\succeq(p,d,q'\theta,0)$, or $s(\sigma+\sigma'+\sigma'')-s(\sigma+\sigma'')\leq \ln\frac{v(u-p(1-d))}{v(u-p'(1-d))}$. Combining the two results yields $s^-(\sigma+\sigma'+\sigma'')-s^-(\sigma+\sigma'')\leq s^-(\sigma+\sigma')-s(\sigma)$. ■

Proof of Proposition 2. **A3.d**. By A6, if $d>d_x$, then $r(d)\leq r(d_x)$. We have that

$$\begin{aligned}(d,x_{-d})=(p_x,d,q_x,t_x) &\sim^{A5}(p_x,d,q_xe^{-r(d)t_x},0)\sim^{A4}(p_x(1-d),0,q_xe^{-r(d)t_x},0)\\ &\succ^{A3.p}(p_x(1-d_x),0,q_xe^{-r(d)t_x},0)\succ^{A3.q}(p_x(1-d_x),0,q_xe^{-r(d_x)t_x},0)\\ &\sim^{A4}(p_x,d_x,q_xe^{-r(d_x)t_x},0)\sim^{A5}(p_x,d_x,q_x,t_x)=x.\end{aligned}$$

That higher discounts are better also holds for $x\prec\mathbf{0}$ with $t_x=0$. It also holds for all x, provided $(d,x_{-d})\succ\mathbf{0}$. It may fail if $t_x>0$ and $(d,x_{-d})\prec\mathbf{0}$: increasing a bit the price discount of an unattractive item could make the future purchase even less attractive because the future loss becomes more salient.

A7.t. Let $x\sim y\succ\mathbf{0}$ with $t_x\geq t_y$, $d_x\geq d_y$, and $q_x=q_y=q$. Note that $\theta'=e^{-r(d_x)(t_x-t_y)}$ solves $x\sim(t_y,\theta' q,x_{-tq})$. Given $\Delta\geq 0$ in A7.t, let $\theta=e^{-r(d_y)\Delta}$ solve $y\sim(t_y+\Delta,\theta q,y_{-tq})$. Because $r(d_x)\leq r(d_y)$, we have that $e^{-[r(d_y)-r(d_x)]\Delta}\leq 1$. Applying A7.p to y and $(t_y,\theta' q,x_{-tq})$, produces

$$(\theta q,y_{-q})\preceq(t_y,\theta'\theta q,x_{-tq}).$$

By A5, the left prospect is indifferent to $(t_y+\Delta,q,y_{-qt})$. By A5 (twice), the right prospect is indifferent to both $(t_x,\theta q,x_{-tq})$ and $(t_x+\Delta,e^{-[r(d_y)-r(d_x)]\Delta}q,x_{-tq})$. This, and A3.q, implies

$$(t_y+\Delta,y_{-t})\preceq(t_x+\Delta,e^{-[r(d_y)-r(d_x)]\Delta}q,x_{-tq})\preceq(t_x+\Delta,x_{-t}).$$

For $x\sim y\prec\mathbf{0}$, we repeat the same construction except that A7.p yields $(\theta q,y_{-q})\succeq(t_y,\theta'\theta q,x_{-tq})$, and $e^{-[r(d_y)-r(d_x)]\Delta}\leq 1$ implies $(t_x+\Delta,e^{-[r(d_y)-r(d_x)]\Delta}q,x_{-tq})\succeq(t_x+\Delta,x_{-t})$. Hence, A7.t follows. ■

Proof of Proposition 3. If $x \in \mathcal{X}^0$, then $q_x e^{-t_x} = 0$, $q_x e^{-t_x} e^{\tau} = 0$, and $(t_x - \tau, x_{-t}) \in \mathcal{X}^0$. Hence, $(t_x - \tau, x_{-t}) \sim_0 \mathbf{0}$. By A2, $x \sim_\tau \mathbf{0}_\tau$.

$x \succ_0 \mathbf{0}$ and A3.t implies $(t_x - \tau, x_{-t}) \succ_0 x$. By A2, $(t_x - \tau, x_{-t}) \succ_0 \mathbf{0}$ implies $(t_x, x_{-t}) \succ_\tau \mathbf{0}_\tau$.

$x \prec_0 \mathbf{0}$ and A3.t implies $(t_x - \tau, x_{-t}) \prec_0 x$. By A2, $(t_x - \tau, x_{-t}) \prec_0 \mathbf{0}$ implies $(t_x, x_{-t}) \prec_\tau \mathbf{0}_\tau$.

Once the consumer learns at time t that the product is available, the prospect (p, d', q, t) becomes $(p, d', 1, t)$. By A4pd, A3.p, and our premise, respectively,

$$(p, d', 1, 0) \sim_0 (p(1-d'), 0, 1, 0) \succ_0 (p(1-d), 0, 1, 0) \succ_0 (p, d, 1, 0) \succ_0 \mathbf{0}.$$

By time invariance, if $(p, d', 1, 0) \succ_0 \mathbf{0}$, then $(p, d', 1, t) \succ_t \mathbf{0}_t$. ■

Proof of Proposition 4. By definition, the best response threshold is equal to $\mathcal{B}(H) = 1$ if $V_0(p, d, q_2, t) \geq V_0(p, 0, q_1, 0)$ for all $u \in [p, 1]$. Otherwise is given by the value of u that makes 'wait' and 'buy now' indifferent, i.e., the solution to $V_0(p, d, q_2, t) = V_0(p, 0, q_1, 0)$. Solving for u yields the expression inside the bracket of (7).

To see that $\mathcal{B}(H)$ is increasing in H, we distinguish three cases.

- If $Q \leq \lambda_1$, then $q_1 = Q/\lambda_1$, $q_2 = 0$, $\mathcal{B}(H) = p$, and $\mathcal{B}'(H) = 0$.
- If $Q \geq \lambda_1 + \lambda_2$, then $q_1 = q_2 = 1$, $dq_1/dH = dq_2/dH = 0$ and $\mathcal{B}'(H) = 0$.
- If $Q \in (\lambda_1, \lambda_1 + \lambda_2)$, then $q_1 = 1$, $q_2 = (Q - \lambda_1)/\lambda_2 \in (0, 1)$,

$$\begin{aligned}\frac{\partial \mathcal{B}(H)}{\partial q_2} &= pd \frac{s'\left(\ln 1/q_2 + r(d)t\right) e^{-s(\ln 1/q_2 + r(d)t)}}{q_2 [1 - e^{-s(\ln 1/q_2 + r(d)t)}]^2} > 0, \\ \frac{dq_2}{dH} &= (\lambda_2 + \lambda_1 - Q) f(H) \lambda / \lambda_2^2 > 0,\end{aligned} \tag{17}$$

and $\mathcal{B}'(H) = \partial \mathcal{B}(H)/\partial q_2 \cdot dq_2/dH > 0$.

Note that $Q \leq \lambda_1$ is equivalent to $H \leq F^{-1}\left(\max\{\lambda - Q, 0\}/\lambda, 0\right)$ and equivalent to $q_2 = 0$. Note also that $\lambda_1 + \lambda_2 = \lambda \bar{F}(p(1-d))$, which is independent of H. ■

Proof of Proposition 5. Suppose consumers play $H \in [p, 1]$.

Regime I. Consumers enter the market if and only if $u \geq p(1-d)$. Hence, $\lambda \bar{F}(p(1-d)) = \lambda_1 + \lambda_2$ is the number of consumers that enter. If $Q > \lambda_1 + \lambda_2$, then all consumers purchase with probability one. Plugging $q_1 = q_2 = 1$ into (7) and using $s(0) = 0$ yields $\mathcal{B}(H)$ equal to (8). Clerarly, $H^* = \mathcal{B}(H)$ is the unique solution to $H^* = \mathcal{B}(H^*)$. We verify that $\mathcal{B}(H) \in (p, 1]$.

Regime II. Because $Q > \lambda \bar{F}(p)$ and $H \geq p$, it follows that $\bar{F}(H) \leq \bar{F}(p)$ and $Q > \lambda \bar{F}(H) = \lambda_1$. Thus, all consumers in period 1 are served, and the quantity available in period 2 is $Q - \lambda_1 > 0$. That $Q < \lambda \bar{F}(p(1-d))$ implies $Q - \lambda_1 < \lambda \bar{F}(p(1-d)) - \lambda_1 = \lambda_2$. It follows that $q_2 = (Q - \lambda_1)/\lambda_2 \in (0, 1)$. That $q_2 > 0$ implies $\mathcal{B}(H) > p$ and $H^* > p$.

Regime III. Suppose consumers play according to $H = p$. Then $q_1 = \frac{Q}{\lambda \bar{F}(p)} < 1$ and $q_2 = 0$. For $u < p$, the best response does not depend on H (consumers with $p(1-d) \leq u < p$ will surely wait). For $u \geq p$, the value of the buy-now option is non-negative because $q_1 \geq 0$, whereas the value of the wait option is strictly zero because $q_2 = 0$. Thus, 'buy now' is best and $\mathcal{R}(p) = p$.

To show that $H^* = p$ is the only equilibrium with $q_1 < 1$, assume there is an equilibrium with $H^* > p$ and $q_1^* < 1$. This implies $Q < \lambda_1$ and $q_2^* = 0$. For $u \in [p, H)$, equilibrium specifies to 'wait', but 'buy now' is better because $V_0(p, 0, q_1, 0) > V_0(p, d, 0, t) = 0$, a contradiction. ■

Proof of Proposition 6. We show that all consumers are weakly better off under H.

- Consumers with $u \in [H, 1]$ always choose 'buy now'. In regime II they obtain the item with $q_1 = 1$ and are equally off. Under regime III they are weakly better off under H because H' may be equal to p and exhibit $q_1' < 1$, whereas H is surely higher than p and necessarily yields $q_1 = 1$.
- Consumers with $u \in [H', H)$ would choose 'buy-now' under H' because $V_0(p, 0, 1, 0) \geq V_0(p, d, q_2', t)$; and choose 'wait' under H because $V_0(p, d, q_2, t) > V_0(p, 0, 1, 0)$. The latter condition also indicates that they are better off under H than under H'.
- Consumer with $u \in [p(1-d), H']$ always choose 'wait', but obtain a larger payoff in the equilibrium with highest value of q_2, which is H.
- Consumers with $u < p(1-d)$ would choose 'opt-out' in both equilibrium and are equally off.

Hence, H Pareto-dominates H'. It follows that if the equilibrium with highest q_2 also has the highest H, then this equilibrium is Pareto-dominant. ■

Proof of Proposition 7. We prove a more general version of the result using the general dPTT formulation with $r(d) = \rho = const$. Note $r(d) = const$ corresponds to $\mu = 0$ in (3). Recall that DEU is a special case of dPTT with $s(\sigma) = \sigma$, and $r(d) = const$.

Suppose (and verify later) that $Q > \lambda\left(1 - \frac{1}{2}p(1 + e^{-\rho t})\right)$ implies no rationing at the optimal discount, i.e., $q_1 = q_2 = 1$. Then

$$R^{DEU} = \lambda p \left(1 - F(H^{DEU}) + (1-d)(F(H^{DEU}) - F(p(1-d)))\right) = \lambda p(1 - H^{DEU} + (1-d)(H^{DEU} - p(1-d))).$$

First order optimality condition:

$$\frac{\partial R^{DEU}}{\partial d} = 0, \text{ or } H^{DEU} + d\frac{\partial H^{DEU}}{\partial d} + 2p(d-1) = 0,$$

where $H^{DEU} = p\frac{1-(1-d)e^{-\sigma}}{1-e^{-\sigma}}$, $\sigma = \rho t$, and $\frac{\partial H^{DEU}}{\partial d} = p\frac{e^{-\sigma}}{1-e^{-\sigma}}$.

Therefore:

$$\frac{1-(1-d)e^{-\sigma}}{1-e^{-\sigma}} + d\frac{e^{-\sigma}}{1-e^{-\sigma}} + 2(d-1) = 0,$$

$$1+\frac{2de^{-\sigma}}{1-e^{-\sigma}}+2d-2=0, \text{ or } d\triangleq d^{DEU}=\frac{1}{2}\left(1-e^{-\sigma}\right).$$

At the optimal discount level d^{DEU}, the cumulative demand over two periods is $\lambda(1-p(1-d))=\lambda\left(1-\frac{1}{2}p(1+e^{-\sigma})\right)$, $\sigma=\rho t$, hence condition of the proposition verifies that there is indeed no rationing.

Consider the dPTT case. The derivative of the revenue function:

$$\frac{\partial R^{dPTT}}{\partial d}=-\frac{\partial(dH^{dPTT})}{\partial d}-2p(d-1),$$

where $H^{dPTT}=p\frac{1-(1-d)e^{-s(\sigma)}}{1-e^{-s(\sigma)}}$. Substituting $\frac{\partial(dH^{dPTT})}{\partial d}=\frac{p}{1-e^{-s(\sigma)}}\left(1-e^{-s(\sigma)}(1-2d)\right)$, obtain

$$\frac{\partial R^{dPTT}}{\partial d}=\frac{p}{1-e^{-s(\sigma)}}\left(e^{-s(\sigma)}(1-2d)-1\right)-2p(d-1).$$

At $d=d^{DEU}$:

$$\begin{aligned}\frac{\partial R^{dPTT}}{\partial d}\Big|_{d=d^{DEU}}&=\frac{p}{1-e^{-s(\sigma)}}\left(e^{-s(\sigma)}e^{-\sigma}-1\right)+p(1+e^{-\sigma})\\&=\frac{p}{1-e^{-s(\sigma)}}\left(e^{-s(\sigma)}e^{-\sigma}-1+(1+e^{-\sigma})(1-e^{-s(\sigma)})\right)\\&=\frac{p}{1-e^{-s(\sigma)}}\left(e^{-\sigma}-e^{-s(\sigma)}\right)>0 \text{ if } s(\sigma)>\sigma.\end{aligned}$$

Since $\sigma=\rho t<1$, s – concave with $s(0)=0$ and $s(1)=1$, $s(\sigma)>\sigma$ ■.